ON A DISTANCE REPRESENTATION OF KANTOROVICH POTENTIALS

LUCA GRANIERI

ABSTRACT. We address the question to represent Kantorovich potentials in mass transportation (or Monge-Kantorovich) problem as a signed distance function from a closed set. We discuss geometric conditions on the supports of the measure f^+ and f^- in the Monge-Kantorovich problem which ensure such representation. Finally, as a by-product, we obtain the continuously differentiability of the potential on the transport set.

Keywords. Monge-Kantorovich problem, mass transportation problems, maximal transport rays.
MSC 2000. 49Q20, 49N60, 74P20.

The Monge-Kantorovich problem. Assume that we are given a pile of soil and an excavation that we want to fill up with the soil. In 1781 Monge posed the question to find an optimal way to do this. We can model the pile of soil and the excavation by two probability measures $f^+, f^- \in \mathcal{P}(\Omega)$ over a given open and bounded set $\Omega \subset \mathbb{R}^N$. We denote by $|\cdot|$ the euclidean norm on $\mathbb{R}^N$.

We consider a measurable map $t : \Omega \to \Omega$ as a *transport* between f^+ and f^- if the amount of mass of f^- on a region B of Ω is the same coming from f^+ through the map t. Hence, if we consider a Borel set $B \subset \Omega$ we require that $f^-(B) = f^+(t^{-1}(B))$. In other words we have that f^- is the image measure of f^+ through the map t. We use the notation $t_\sharp f^+ = f^-$ (push-forward of measures) whenever the previous condition holds. If $|x-y|$ is the cost to move the particle in x to the position y, Monge problem can be written as follows:

$$\inf\left\{\int_\Omega |x-t(x)|\, df^+ \mid t_\sharp f^+ = f^-\right\}, \tag{1}$$

where the unknown is the transport map t.

Observe that the Monge problem is not always well posed. In fact if we consider for example the measures $f^+ = \delta_x$ and $f^- = \frac{1}{2}(\delta_y + \delta_z)$, the Monge transport problem has no solutions simply because there is no map t such that $t_\sharp f^+ = f^-$. Moreover, because of the non-linearity of the cost with respect to t, existence of minimizers in (1) is a difficult matter and the first rigorous existence theorems are relatively recent, see ([1, 5, 8, 11]), despite the long history of the problem. In order to avoid these difficulties the problem can be reformulated in its Kantorovich relaxed form. If π_1, π_2 are the projections of $\Omega \times \Omega$ on his factors and $|x-y|$ is the cost to move the particle in x to the position y, the Monge-Kantorovich problem amounts to

$$\min\left\{\int_{\Omega\times\Omega} |x-y|\, d\gamma(x,y) \mid \gamma \in \mathcal{P}(\Omega\times\Omega)\ , (\pi_1)_\sharp\gamma = f^+,\ (\pi_2)_\sharp\gamma = f^-\right\}. \tag{2}$$

The admissible measures γ for problem (2) are called transport plans. Observe that if t is admissible for the Monge problem then the measure $\gamma = (id \times t)_\sharp f^+$ is a transport plan for (2). Furthermore, the class of transport plans is never empty as it always contains $f^+ \otimes f^-$. Note that the Kantorovich problem (2) is now linear and existence is quite easy to obtain.

An important step to treat Monge problem is a dual formulation due to Kantorovich.

Theorem 1. *For every $f^+, f^- \in \mathcal{P}(\Omega)$ the minimum value of the Monge-Kantorovich problem (2) is equal to*

$$\max\left\{\int_\Omega u\, d(f^+ - f^-) \mid u \in \mathrm{Lip}_1(\Omega)\right\},$$

where $\mathrm{Lip}_1(\Omega)$ *denotes the set of Lipschitz functions with Lipschitz constant not greater than* 1.

The Lipschitz functions u for which the maximum in Theorem 1 is attained are called Kantorovich potentials. The existence of Kantorovich potentials u is important since, roughly speaking, they determine the directions, given by Du, and then the segments (transport rays) for moving the masses (see the next section for more details). The notion of transport ray was introduced by Evans and Gangbo in [8]. In particular, in their PDE approach to the transportation, Evans and Gangbo derived the system of equations

$$\begin{cases} -div(\sigma Du) = f^+ - f^- \\ |Du| = 1 \quad \sigma - a.e. \end{cases} \tag{3}$$

where $u \in \mathrm{Lip}_1(\Omega)$ is a Kantorovich potential, while σ is a measure called transport density. Actually, roughly speaking, the measure σ establishes the amount of mass to move along a transport ray whose direction is given by Du. The equations in (3) are important in many different contexts, such as shape optimization and granular matter theory. For theory and applications we refer to [3, 4, 7]. Denoting by $\mu = f^+ - f^-$, if $\mu \geq 0$, then the equations (3) model a quite different problem arising in the study of equilibrium solutions for growing sandpiles as treated for example in [6, 10]. In particular in [6, 10] it is shown that the system of equations

$$\begin{cases} -div(\sigma Du) = \mu \quad \text{in } \Omega \\ |Du| = 1 \quad \sigma - a.e. \\ u = 0 \quad \text{on } \partial\Omega \end{cases} \tag{4}$$

admits a unique solution (σ, u) with $u(x) = dist(x, \partial\Omega)$, where for every $A, B \subset \mathbb{R}^N$ we denote $dist(A, B) = \inf\{|x - y| \mid x \in A, y \in B\}$. Moreover, they provide a representation formula for σ in terms of the distance function $dist(\cdot, \partial\Omega)$. Therefore, these results can be regarded as regularity results for the equations (4).

Description of the results. The aim of this paper is to investigate if, also in the general case of the transport problem, one can have a Kantorovich potential given by a distance function. Actually, there are several regularity results for the transport density σ which appears in (3), see for example [7]. However, since in general the transport density σ is merely a measure, some regularity on u is necessary to give meaning to the pairing σDu. In this paper we address the question to represent a Kantorovich potential u by a signed distance on the transport set, i.e. $u(x) = \pm d(x, \Gamma)$, where Γ is a suitable closed set. Actually, this representation holds if it is possible to prescribe the change of sign of the potential u on the supports M of f^+ and N of f^- (see Lemma 9). In fact, by well known properties of the Kantorovich potentials u, it follows that u behaves locally as a signed distance function from each level set of u. On the other hand, it is clear that in general u cannot behave globally as a signed distance simply because M and N could have mutual position which prevent a 1-Lipschitz function from having the correct sign. In Theorem 7 we states the representation by a signed distance of Kantorovich potentials whenever the supports M, N are sufficiently far. If this condition does not hold, we provide a counterexample in which none of the potentials can be represented by a signed distance function. In particular, this phenomenon occurs also if the supports M and N are separated by a positive distance (see Example 8). The hypothesis of separated supports has been already used several times, see for instance [8, 9], to obtain additional properties of the Kantorovich potentials. However, to recover the distance representation of the potentials one need some more assumptions, as the smallness of the diameters of the supports with respect to the separation of the supports themselves (see Corollary 9). Finally, as a by-product, we use these results to obtain the continuously differentiability of the Kantorovich potentials on the transport set (see Corollary 10).

Geometry of transport rays. Kantorovich potentials are an important tool in all the existence proofs for the Monge problem that are available at the moment.

In the rest of the paper we take Ω a convex, open and bounded set of $\mathbb{R}^n$. A key lemma to find optimal transport map for problem (1) is the following (Lemma 6 in [5]).

Lemma 2. *Let $u \in \mathrm{Lip}_1(\Omega)$, and $t : \Omega \to \Omega$ be a Borel map such that $t_\sharp f^+ = f^-$. Then, u is a Kantorovich potential and t is an optimal transport map for problem (1) if and only if*

$$u(x) - u(t(x)) = |x - t(x)| \quad \text{for } f^+ - a.e.\ x \in \Omega. \tag{5}$$

Condition (5) contains a useful geometric meaning. In fact, let $x, y \in \Omega$ such that

$$u(x) - u(y) = |x - y|. \tag{6}$$

If $z \in [x, y]$, since $u \in \mathrm{Lip}_1(\Omega)$ we have

$$|y - z| = |x - y| - |x - z| = u(x) - u(y) - |x - z| \leq u(x) - u(y) + u(z) - u(x) \Rightarrow$$

$$\Rightarrow u(z) = u(y) + |z - y|.$$

Therefore, the function u is decreasing with the maximum possible rate along the segment $[x, y]$. Furthermore, by the triangular inequality we find that for every $z_1, z_2 \in [x, y]$ it also results $u(z_1) - u(z_2) = |z_1 - z_2|$. Indeed

$$u(z_1) - u(z_2) = u(z_1) - u(y) + u(y) - u(z_2) = |z_1 - y| - |z_2 - y| = \pm|z_1 - z_2|.$$

For reader's convenience we state the following important well known property (see for instance [12]) of Kantorovich potentials.

Lemma 3. *Let u be a Kantorovich potential. Then the following condition holds*

$$\begin{aligned} &\forall y \in N : u(y) = u_*(y) := \max\{u(x) - |x - y| \mid x \in M\}, \\ &\forall x \in M : u(x) = u^*(x) := \min\{u(y) + |x - y| \mid y \in N\}, \end{aligned} \tag{7}$$

where M, N are the supports of f^+, f^- respectively.

Proof. Observe that the function $u^*, u_* \in \mathrm{Lip}_1(\Omega)$ and $u_* \leq u \leq u^*$. Moreover u_*, u^* are respectively the smallest and the largest 1-Lipschitz extension of u outside M, N since it is immediate to check that $u = u_*$ in M and $u = u^*$ in N. Suppose now by contradiction that $u(x) < u^*(x)$ for some $x \in M$. By continuity there exists a small radius $r > 0$ such that $u < u^*$ in $B(x, r)$. Since $x \in M$ we deduce $f^+(B(x, r)) > 0$ and this implies $\int_M u \, df^+ < \int_M u^* \, df^+$. Therefore we get

$$\int_\Omega u \, d(f^+ - f^-) = \int_M u \, df^+ - \int_N u \, df^- < \int_M u^* \, df^+ - \int_N u^* \, df^- = \int_\Omega u^* \, d(f^+ - f^-)$$

and this contradicts the maximality of u. The other equality $u = u_*$ in N follows in a similar way. □

If u is a Kantorovich potential, any transport map moves the mass along the segments determined by the condition (6) with $x \in M$ and $y \in N$. We will call these segments transport rays. The precise definition is the following.

Definition 4 (Transport rays)**.** *A transport ray $R_{x,y}$ is a segment joining x and y such that*
1) $x \in M,\ y \in N, x \neq y$,
2) $u(x) - u(y) = |x - y|$,
3) Maximality: set $a_t = x + t(y - x)$. Then for any $t < 0$ such that $a_t \in M$ we have $|u(a_t) - u(y)| < |a_t - y|$, and for any $t > 1$ such that $a_t \in N$ we have $|u(a_t) - u(x)| < |a_t - x|$.

We call the points x, y the upper and lower ends of $R_{x,y}$ respectively. Hence, condition (5) asserts that any transport map moves the mass along the transport rays. We remark that by the relations (7) we have that every point on M, N belongs to some transport ray. Furthermore, the data f^+, f^- are supported on the transport rays. We denote by T (transport set) the union of all transport rays. Another basic observation is that transport rays do not cross, according to the following (Lemma 10 in [5])

Lemma 5. *Let $R_1 \neq R_2$ be two transport rays. If $R_1 \cap R_2 = \{c\}$ then c is either the upper end of both rays or the lower end of both rays. In particular, an interior point of a transport ray does not lie in any other transport ray.*

Distance representation of Kantorovich potentials. In this section we address the question whether or not it is possible to represent a Kantorovich potential by a signed distance function. The crucial condition is the the change of sign of the Kantorovich potential on the supports M, N.

Lemma 6. *Let u be a Kantorovich potential such that $u \geq 0$ on M and $u \leq 0$ on N. Setting $\Sigma_0 = \{x \in \Omega \mid u(x) = 0\}$, $T^+ = \{x \in T \mid u(x) > 0\}$, $T^- = \{x \in T \mid u(x) < 0\}$, it results*

$$u = \begin{cases} dist(\cdot, \Sigma_0) & on\ T^+ \cup \Sigma_0, \\ -dist(\cdot, \Sigma_0) & on\ T^- \cup \Sigma_0. \end{cases} \tag{8}$$

Proof. The potential u satisfies the following inequalities:

$$\forall x \in T^+ \cup \Sigma_0 : dist(x, \Sigma_0) \leq u(x)\ ,\ \forall y \in T^- \cup \Sigma_0 : dist(y, \Sigma_0) \leq -u(y).$$

Indeed, for every $x \in T^+ \cap \Sigma_0$, if $u(x) = 0$ then $dist(x, \Sigma_0) = 0$. In the case $u(x) > 0$, consider a transport ray $R_{a,b}$ such that $x \in R_{a,b}$. Hence $u(x) = u(b) + |x - b|$. If $u(b) = 0$, then $dist(x, \Sigma_0) \leq |x - b| = u(x)$. Otherwise, by continuity of u along the transport ray, there exists $z \in R_{a,b}$ such that $u(z) = 0$. Then again $dist(x, \Sigma_0) \leq |x - z| = u(x)$. The other inequality follows in a similar way. On the other hand, for every $y \in \Sigma_0$ it results $|u(z)| \leq |z - y|$ and this implies $|u(z)| \leq dist(z, \Sigma_0)$. Therefore $u(x) = dist(x, \Sigma_0)$ on $T^+ \cup \Sigma_0$ and $u(y) = -dist(y, \Sigma_0)$ on $T^- \cup \Sigma_0$. □

We will say that a potential u is representable by a signed distance function whenever condition (8) holds. The rest of this section is devoted to discuss some geometric conditions in order to have the distance representation (8) for Kantorovich potentials. We remark that since f^+, f^- are both probability measures, the Kantorovich potentials are determined up to addition of a constant. Therefore, given a Kantorovich potential u, by adding a constant one can always assume the sign of u prescribed on $spt(f^+) = M$. The difficulty is then to control the sign of u on the other support $spt(f^-) = N$. An assumption which ensures the condition (8) is given by considering supports M, N sufficiently far. We denote by $diam(A) = \sup\{|x - y| \mid x, y \in A\}$ the diameter of $A \subset \mathbb{R}^N$.

Theorem 7. *Let u be a Kantorovich potential. If*

$$dist(M, N) \geq \min\left(diam(M), diam(N)\right), \tag{9}$$

then, up to addition of a constant, u is representable by a signed distance function.

Proof. Suppose that $dist(M, N) \geq diam(M)$ and let $m = \min_{x \in M} u(x)$. Adding a constant to u we can assume that $m = 0$, so that $u \geq 0$ on M. Fix $x_0 \in M$ such that $u(x_0) = 0$. Hence, for every $x \in M$ we have $u(x) = u(x) - u(x_0) \leq |x - x_0| \leq diam(M)$. By (7) and (9), for every $y \in N$ we have

$$u(y) = \max_{x \in M}\{u(x) - |x - y|\} \leq diam(M) - dist(M, N) \leq 0.$$

We obtain the same conclusion arguing in a similar way if $dist(M, N) \geq diam(N)$. Then the result follows by Lemma 6. □

The condition (9) is not a necessary condition. Indeed, consider the following measures supported on the real line

$$f^+ = \frac{1}{2}\delta_1 + \frac{1}{2}\delta_3, \quad f^- = \frac{1}{2}\delta_0 + \frac{1}{2}\delta_2.$$

Therefore we have

$$dist(M, N) < \min(diam(M), diam(N)).$$

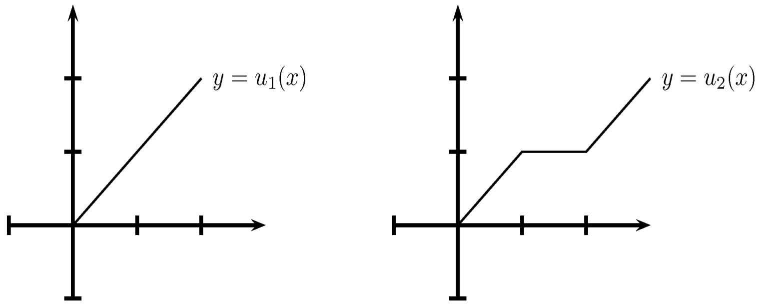

FIGURE 1. Two different Kantorovich potentials.

The Kantorovich potentials are uniquely determined, up to addition of a constant, on transport rays, where they increase with maximum rate. For the measures above considered, the transport rays correspond to the segments connecting the Dirac deltas of f^- with those of f^+. In Figure 1 we have two different Kantorovich potentials u_1, u_2. It turns out that $u_i - u_i(1)$, $i = 1, 2$, is representable by a signed distance. However, the situation is more involved since there are also distributions of masses for which none of the potentials can be represented by a signed distance function.

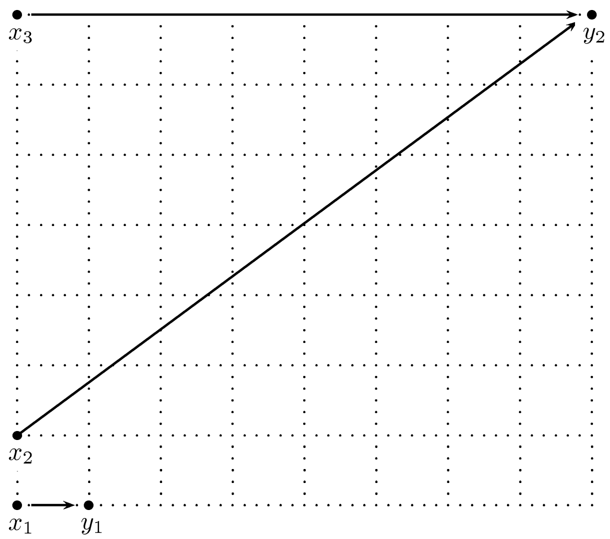

FIGURE 2. The arrows represent the transport rays and their directions.

Example 8. Consider the following measures supported on the plane

$$f^+ = \frac{1}{2}\,\delta_{x_1} + \frac{1}{4}\,\delta_{x_2} + \frac{1}{4}\,\delta_{x_3}, \quad f^- = \frac{1}{2}\,\delta_{y_1} + \frac{1}{2}\,\delta_{y_2},$$

where we set $x_1 = (0, 0), x_2 = (0, 1), x_3 = (0, 7), y_1 = (1, 0), y_2 = (8, 7)$. It results that the optimal transport map t is given by $t(x_1) = y_1,\ t(x_2) = y_2,\ t(x_3) = y_2$. Let u be a Kantorovich potential. Since $|x_1 - y_1| = 1 = |x_2 - x_1|, \quad |x_3 - x_2| = 6, \quad |x_3 - y_2| = 8$, we have

$$u(x_3) = u(y_2) + |x_3 - y_2| = u(x_2) - |x_2 - y_2| + |x_3 - y_2| \le u(y_1) + |x_2 - y_1| - |x_2 - y_2| + |x_3 - y_2|.$$

Therefore, if we suppose that $u(y_1) \leq 0$ then we would have $u(x_3) < 0$. Hence the representation (8) by a signed distance does not hold for u.

Example 8 shows that the representation of potentials by a signed distance does not hold also if the supports are separated by a positive distance. Actually, in order to recover the representation property (8) one need some more assumptions, as the smallness of the diameters of the supports with respect to the separation of the supports themselves.

Corollary 9. *Let u be a Kantorovich potential. Suppose that there exists $\lambda \in \mathbb{R}^N, \lambda \neq 0, \alpha \in \mathbb{R}$ such that the following separation property holds*

$$\forall x \in M, \forall y \in N : \lambda \cdot x \leq \alpha \leq \lambda \cdot y. \tag{10}$$

If the supports M, N satisfy the condition

$$diam(M) \leq \min_{x \in M} \frac{1}{|\lambda|}(\alpha - \lambda \cdot x) \ , \ diam(N) \leq \min_{y \in N} \frac{1}{|\lambda|}(\lambda \cdot y - \alpha) \tag{11}$$

then, up to addition of a constant, u is representable by a signed distance.

Proof. Let $x \in M, y \in N$. By Cauchy-Schwarz inequality and conditions (10) and (11) we have

$$|x - y| \geq (y - x) \cdot \frac{\lambda}{|\lambda|} = \frac{1}{|\lambda|} \left(y \cdot \lambda - \alpha + \alpha - x \cdot \lambda \right) \geq diam(M) + diam(N).$$

Taking the infimum with respect to x, y we get

$$dist(M, N) \geq min(diam(M), diam(N)).$$

Hence, the result follows by Theorem 7. □

Differentiability on the transport set. By Rademacher theorem every Kantorovich potential is differentiable almost everywhere. It is not hard to see that if we denote by T_0 the union of all points which lie in the interior of some transport ray, then a Kantorovich potential u is in fact differentiable on T_0 (Lemma 4.1 in [8]). Actually, since the potential u studied in [8] is obtained by a p-Laplacian approximation, it turns out that u satisfy some semiconvexity properties and then it is continuously differentiable in $T_0 \backslash (M \cup N)$. Here we use the results of the previous section to obtain the continuously differentiability of any potential on T_0. Observe that semiconvexity properties follow by standard property of the distance function. For a proof which relies on Lagrangian dynamic see [2].

Corollary 10. *If there exists $\varepsilon > 0$ such that the supports M and N verify the following condition*

$$dist(M, N) \geq \varepsilon + \min\left(diam(M), diam(N)\right), \tag{12}$$

then any Kantorovich potential u is continuously differentiable in T_0.

Proof. Following the construction made in the proof of Theorem 7, adding a constant to u (namely taking $min_{x \in M}\ u(x) = \varepsilon/2$) we may assume that $u \geq \varepsilon/2$ in M and $u \leq \varepsilon/2$ in N. Therefore, u is representable by a signed distance function from the closed set Σ_0, which is the 0-level set of u. Moreover, if we denote by T_e the set of endpoints of all transport rays we get $dist(z, T_e) \geq \varepsilon/2$ for every $z \in T_0 \cap \Sigma_0$. Let $x \in T_0 \setminus \Sigma_0$ and suppose for instance that $u(x) > 0$. By standard properties of the distance function, it is enough to prove that there exist a unique $z_0 \in \Sigma_0$ such that $u(x) = |x - z_0| = dist(x, \Sigma_0)$ in order to check the continuously differentiability of u at the point x. If R_x is the transport ray passing through x, which is unique by Lemma 5, since the Kantorovich potentials increase at rate one on the transport rays, we have that there exist a unique $z_0 \in \Sigma_0 \cap R_x$ such that $u(x) = |x - z_0| = dist(x, \Sigma_0)$. Suppose now by contradiction that there exist another point $z \in \Sigma_0$ such that $|x - z| = dist(x, \Sigma_0) = u(x)$. Since z does not lie on R_x, denoting by a the upper end of R_x, by triangular inequality we have

$$|a - z| < |a - x| + |x - z| = |a - x| + |x - z_0| = |a - z_0|. \tag{13}$$

On the other hand, since $a \in T^+$ it results

$$|a - z_0| = u(a) = dist(a, \Sigma_0) \leq |a - z|$$

which contradicts (13). By standard properties of the distance function (see for example [10]) it follows that u is continuously differentiable on $T_0 \setminus \Sigma_0$. Moreover it turns out that $Du(x) = e$ where $e = \frac{a-b}{|a-b|}$ is the ray direction of R_x. It remains to check what happens on points $z_0 \in \Sigma_0 \cap T_0$. By the arguments performed up to now, it immediately follows that u is also differentiable at z_0. In particular we have $Du(z_0) = e_0$ with e_0 the ray direction of the transport ray passing through z_0. If $x \in T_0$, then there exists a unique $z \in \Sigma_0 \cap R_x$. Moreover we know that $dist(z, T_e) \geq \varepsilon/2$. Therefore we can use Lemma 16 in [5] which states that ray directions vary Lipschitz continuously on the level sets of u. Observing that $|x - z| = |u(x)| \leq |x - z_0|$, by Lemma 16 in [5] we have

$$|Du(z_0) - Du(x)| = |Du(z_0) - Du(z)| \leq K|z_0 - z| \leq K|x - z_0| + K|x - z| \leq 2K|x - z_0|.$$

The above inequality completes the proof.

□

Acknowledgements. The author is grateful to the referees for suggesting substantial improvements to the previous version of this paper.
The author wish also to thank professor G. Buttazzo and L. De Pascale for the constant attention and for useful discussions.

References

[1] L. Ambrosio, Lecture Notes on Transport Problems, in "Mathematical Aspects of Evolving Interfaces", Lecture Notes in Mathematics **1812**, Springer, Berlin, 2003, 1-52.

[2] P. Bernard, B. Buffoni, The Monge problem for supercritical Mañé potentials on compact manifolds, Adv. Math. **207** (2006), no. 2, 691-706.

[3] G. Bouchitté, G. Buttazzo, Characterization of optimal shapes and masses through Monge–Kantorovich equation, Journal European Math. Soc. **3** (2001), no. 2, 139–168.

[4] G. Bouchitté, G. Buttazzo, P. Seppecher, Shape optimization solutions via Monge–Kantorovich equation, C. R. Acad. Sci. Paris **324-I** (1997), no. 10, 1185–1191.

[5] L. Caffarelli, M. Feldman, R. J. McCann, Constructing optimal maps for Monge's transport problem as a limit of strictly convex costs, J. Amer. Math. Soc. **15** (2002), no. 1, 1-26.

[6] P. Cannarsa, P. Cardaliaguet, Representation of equilibrium solutions to the table problem of growing sand piles, J. Eur. Math. Soc. **6** (2004), no.4, 435-464.

[7] L. De Pascale, A. Pratelli, Regularity properties for Monge transport density and for solutions of some shape optimization problems, Calc. Var. Partial Differential Equations **14** (2002), no. 3, 249–274.

[8] L. C. Evans, W. Gangbo, Differential Equation Methods for the Monge-Kantorovich Mass Transfer Problem, Memoirs AMS, Vol. **137**, no. 653, 1999.

[9] I. Fragalà, M. S. Gelli, A. Pratelli, Continuity of an optimal transport in Monge problem, J. Math. Pures Appl. **84** (2005), 1261-1294.

[10] E. Giorgieri, A boundary value problem for a PDE model in mass transfer theory: representation and regularity results, Ph.D. Thesis in Mathematics, Roma Tor Vergata University, Rome, 2004.

[11] N. S. Trudinger, X. J. Wang, On the Monge mass transfer problem, Calc. Var. Partial Differential Equations, **13** (2001), no. 1, 19-31.

[12] C. Villani, Topics in Optimal Transportation, Graduate Studies in Mathematics **58**, AMS, Providence, RI, 2003.

Dipartimento di Matematica Politecnico di Bari, via Orabona 4, 70125 Bari, Italy
E-mail address: `l.granieri@poliba.it, granieriluca@libero.it`