Errata: "Vanishing viscosity limit of the compressible Navier–Stokes equations with general pressure law"

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1. Definition 1.1: part (iii) of the definition should in fact read as $\mu(U \times \mathbb{R}) \leq 0$, which also agrees with the original formulation of [1, Theorem 3].

2. Proof of Theorem 5.1: the lines under equation (5.5) describe the open set $\mathbb{S}$ as a disjoint union of countably many open intervals. This is true, however the representation of $\mathbb{S}$ the union of sets (z_k, w_k) with $z_k < w_k < z_{k+1}$ is not. The correct way of stating this is:

$$\mathbb{S} = \bigcup_k (z_k, w_k),$$

where, whenever $k \neq k'$, we either have $w_k \leq z_{k'}$ or $w_{k'} \leq z_k$. From this, it follows that

$$\operatorname{supp} \nu \subset \bigcup_k \{(\rho, u) \in \mathbb{H} : [z(\rho, u), w(\rho, u)] \cap [z_k, w_k] \neq \emptyset\} \cup V,$$

which differs subtly from what was originally written in the paper. Nevertheless, it is true that

$$\operatorname{supp} \nu \cap \{(\rho, u) \in \mathbb{H} : w(\rho, u) \in (z_k, w_k) \text{ or } z(\rho, u) \in (z_k, w_k)\} = \emptyset,$$

along with

$$\operatorname{supp} \nu \subset V \cup \bigcup_{k:\rho_k, u_k \in \mathbb{R}} \{(\rho_k, u_k)\},$$

and, with $\alpha_k \in [0, 1]$ and the measure ν_V supported only in V,

$$\nu = \nu_V + \sum_k \alpha_k \delta_{(\rho_k, u_k)},$$

as written. Then, before jumping to the final paragraph of the proof, it must be verified that, for any fixed $s \in \mathbb{R}$, no two distinct elements (ρ_k, u_k) and $(\rho_{k'}, u_{k'})$ of the support of ν belong to the interior of the support of $\chi(\cdot, \cdot, s)$. This can be proved in a few lines by contradiction: if two such elements exist then, in view of the disjointness of the intervals $\{(z_j, w_j)\}_{j \in \mathbb{N}}$, we have $\chi(\rho_k, u_k, s) = \chi(\rho_{k'}, u_{k'}, s) = 0$ and so neither point belongs to $\operatorname{int}(\operatorname{supp} \chi(\cdot, \cdot, s))$.

Details of the additional steps mentioned above can be found in [2, Section 2.8.1], in the proofs of Claims 2.77–2.80.

References

[1] Lions, P.-L., Perthame, B., Tadmor, E., Kinetic formulation of the isentropic gas dynamics and p-systems, *Commun. Math. Phys.* **163** (1994) 415–431.

[2] Schulz, S., *Compensated compactness methods in the study of compressible fluid flow,* PhD thesis, University of Oxford, Oxford (2020).