The storage location assignment problem in a multi-level warehouse under correlated batched orders

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Abstract

We propose a simulated annealing algorithm specifically tailored to optimise total retrieval times in a multi-level warehouse under complex pre-batched picking constraints. Using real data from a picker-to-parts order picking process in the warehouse of an European manufacturer, we show that near optimal storage assignments do not necessarily display features presumed in heuristics, such as clustering of positively correlated items or ordering of items by picking frequency. In an numerical experiment run on more than 4000 batched orders with 1 to 150 items per batch, the storage assignment suggested by the algorithm produces a 21% reduction in the total retrieval time with respect to a frequency-based storage assignment.

Keywords: Heuristics, Logistics, Order picking, Simulated annealing, Storage location assignment

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1. Introduction

Warehouses play a key role in modern supply chains [21] and are a significant cost factor to a company: according to the European Logistics Association/AT Kearney report [20], the capital and operation costs of warehouses represent about 25% of the surveyed companies' logistics costs in 2003, while figures for the USA [35] indicate that warehousing contributed to the total logistics costs with a share of 22%.

Order picking, generally defined as the process of retrieving products from storage in response to a specific customer request, is the most labour-intensive operation in warehouses with manual systems, and a very capital-intensive operation in warehouses with automated systems [25, 53]. Estimates of the percentage of order picking costs on the total warehouse costs range as high as 55% in Drury [19] and Bartholdi et al [5] to 65% in Coyle et al. [12]. For these reasons, warehousing professionals consider order picking as the highest priority area for productivity improvements.

Over the last twenty years, many papers have studied order picking processes and optimal strategies or heuristics to optimise subprocesses such as warehouse layout, storage assignment, order batching, order release method and picker routing. However, there is little published research on how to combine these subprocesses optimally: we mention here [13], which compares the performance of S-shaped and Largest Gap routing heuristics for batches of 3 and 4 items, and [1], which focuses on heuristics for order batching to improve the overall performance of order picking systems. A recent article [50] proposes a simultaneous solution of order batching, batch assignment, sequencing and picker routing.

The contribution of this work is to propose a general method to find near optimal storage assignments for correlated *batched orders* of variable and possibly large size (up to 102 unique parts per batch in our tests), which works for many common routing heuristics. This is achieved via a hand-tailored algorithm based on a simulated annealing combinatorial search, which incorporates variable parameters encoding the routing heuristics. The algorithm is designed for multi-level warehouses, the routing heuristics is based on a warehouse design with wide aisles, however, it can be adapted to narrow aisles.

The real-world situation which motivates our approach is the warehouse of an European manufacturer, where each incoming order is the collection of items needed at one specific point in the assembly line. Therefore, the order must be delivered exactly as the batch it was ordered; it is not allowed to split the batch into several smaller orders and to deliver them separately at different times. It is therefore important not only to reduce the mean picking time per batch, but also to minimise very large picking times for one batch in general, as such outliers cause delays in the production line. Another peculiarity of this warehouse is that every storage location is unique, in particular the upper levels of the warehouse contain distinct items. This means that the upper levels are not used for replenishment, but have to be considered as integral parts of the warehouse. An algorithm which aims to optimise the storage locations in such a multi-level warehouse therefore needs several new variables to correctly model this situation, which we describe in Section 4.

2. Warehouse order picking

In human-operated warehouses, the most common system for order picking is the *picker-to-parts* class, where the (human) order picker walks or drives along the aisles to pick items [14, 55, 56].

Optimizing a *picker-to-parts* picking process means to minimise the total time needed for picking all orders in a given time frame. The precise quantity to minimise is indeed *total retrieval time*, which is defined as the sum of pick and travel time and time due to delays. By using s-shaped routing or other routing rules which make the aisles unidirectional, the delay part (which is usually due to congestion) can be a priori removed from the model. The main components to be minimised are therefore (1) the time needed for travelling the warehouse to collect the items and (2) the time needed to perform one pick. In our analysis we assume that the latter is a constant, its size depending on the level in the warehouse where the item is located. See the recent work [40] for a case where the individual picker productivity is taken into account, i.e. the time needed to perform one pick is variable.

A reduction of the time needed for travelling the warehouse can be achieved by various ansatzes, the most common ones involve solving the *storage location assignment problem*, the *routing problem* or allocating optimal *batched orders* for one tour of the picker.

2.1. Storage assignment policies

Early scientific contributions to the storage location assignment problem in warehouses include a taxonomy of possible storage location assignment policies, where the classification between dedicated storage, randomized storage and class-based storage was introduced, see [22, 23, 29] and the references therein.

In our problem, we deal with a warehouse where the individual pieces used in the production of certain products are stored. Therefore, the orders arrive already pre-batched, according to the necessity of the assembly line. The order batches are quite large (on average 30 items per batch in the data we used for testing our algorithm). We therefore seek a solution to the storage location assignment problem for a warehouse with *dedicated storage*-policy under different, but given routing for such pre-batched orders.

The *dedicated storage* policy assigns a fixed location to each product, meaning that this location is reserved even for products that are out of stock. While this may result in a low space utilisation, which may imply higher maintenance costs and possibly longer routing times, dedicated storage locations are an advantage in warehouses for production units, since every stored item is needed regularly.
Moreover, dedicated storage locations help to increase the orientation of the order pickers, leading to an increased routing velocity and fewer wrong picks. Dedicated storage policies allow for logical grouping of storage items, which are often advantageous in retail warehouses [16] and in general for items of very different weight, which can be stored in decreasing weight along the standard picking route, implying a good stacking sequence.
However, as pointed out in the survey [15], analytical models for optimising dedicated storage assignment in manual-pick order-picking systems are still lacking. Existing studies (e.g. [43]) mainly focus on *random storage policies*, which reduce the total space required. This does not automatically improve travel times, as travel distance might increase [10]. It needs a computer-controlled environment to be efficient, as a lack of automation or technical equipment assisting the picking process can lead to slow travelling times due to disorientation and higher percentages of picking errors.
It was observed [54, 55] that a separation of the pick stock (forward area) from the bulk stock (reserve area) can lead to a significant improvement of picking times: in the forward area, a dedicated storage policy is applied, while the bulk area can follow a random storage policy. In this way, the advantages of dedicated storage still hold and disadvantages are reduced. Indeed, this policy is already adapted in many warehouses attached to production units, such as the one in our reference case.
At the interface between research and industry, several papers, such as [6, 7, 9, 18, 37], describe algorithms that solve the storage location assignment problem with real-life constraints. In particular the paper [9] also deals with a multi-level warehouse situation, pointing out that the storage assignment systems need to reflect the structure of the orders.

2.2. Routing policies

The problem of optimal routing for order picking classifies as a Steiner Traveling Salesman Problem [52, 49], which is in general not solvable in polynomial time. However, for a special warehouse aisle configuration, Ratliff and Rosenthal [45] showed that there does exist an algorithm that can solve the problem in running time linear in the number of aisles and the number of pick locations. This algorithm was extended to other situations in [17, 47, 48]. Also, storage assignment and order batching have an impact on the performance of the routing method [44, 51].

However, algorithms are not yet available for every specific layout, and there remains the unsolved problem of aisle congestion by pickers following different routes, and the fact that pickers may deviate from routes that they deem illogical [24]. Because of this, the problem of routing order pickers is mainly solved by using a heuristic, such as the s-shape method. In [30], a routing strategy for a warehouse with U-shaped layout has been introduced and proven to be more efficient under certain conditions. Most heuristic methods for routing order pickers in single-block warehouses assume that the aisles of the warehouse are narrow enough to allow the order picker to retrieve products from both sides of the aisle without changing position [28, 43, 46]. A polynomial-time algorithm for routing order pickers in wide aisles was proposed in [26].

One of the strengths of the algorithm we propose here is that it gives efficient heuristics both for narrow and wide aisles: the routing strategy, the right-hand rule and the aisle length are incorporated as variable parameters and can easily be changed, see formula (4.3).

2.3. Simulated annealing

The *simulated annealing* (SA) method is an optimisation algorithm introduced by Kirkpatrick [36] and independently by Černý [8], which allows to find a near optimal or optimal configuration of atoms (or, in our case, storage items) by changing the configuration while subsequently decreasing the temperature of the system.

In our case, we set the total retrieval time in a warehouse as the cost function which is to be minimised. In an iterative procedure (called *update*), storage containers are exchanged with each other at random (i.e. the *configuration* is changed), leading to an increase or decrease in the cost function. The new configuration is then either accepted or refused via a Metropolis criterion [41], allowing the algorithm to decrease the cost function without getting stuck in local minima.

Simulated annealing algorithms are nowadays widely used in economics, for example in packaging problems [27], the production scheduling problem [38], the corridor allocation problem [2, 3] and solving the storage location assignment problem [4, 42]. A simulated annealing heuristics to minimise total retrieval time involving order batching and sequencing was introduced in [34]. Their algorithm uses a geometric cooling schedule [11] also adapted in [39]. The algorithm proposed in [39] uses a single flexible heuristic based on random moves in a structured manner, in comparison to multiple deterministic neighbourhood search heuristics, as often found in the literature, and is comparably very fast.

Recent results [32] show that solutions obtained by *simulated annealing*, *Iterated Local Search* or the *Attribute-Based Hill Climber* [33, 57] may allow order picking systems to operate more efficiently compared to those obtained with standard constructive heuristics such as the *Earliest Due Date rule*.

3. Problem description and modelling

We consider a multi-level warehouse, storing the items needed in the production line of a European manufacturer. The manufacturer offers his customers to choose optional features when ordering one of the (very few) main products. Due to a high number of possible features, the product is individually manufactured for the customer, and no pre-assembly steps are possible before the customization steps.
Incoming orders to the warehouse consist of large batches of individual items, which are picked manually into a bin using a picker-to-parts order picking system and delivered to several output locations. The first two levels of the warehouse are easily accessible, while all levels above can only be accessed by a lifting device, resulting in a higher picking time per item.
An important feature of our problem, given as a hard constraint by our industrial partner, is that the batch size is fixed, as each batch corresponds to the parts used in one step of the assembly line of the production site. This means in particular that the whole batch has to arrive at the assembly line simultaneously, the delay of only one item will result in delaying the whole production. The significance of our batches is therefore very different from the literature [31]: we are not allowed to optimise the batch size, their composition and size is given input data for us.
It is easy to see now that the items in one batch are correlated to each other, as they correspond to a specific production step, i.e. the mounting of the left mirror of a car (in the case of a car manufacturer). In particular, the correlations between individual items to be picked are quite complex and cannot be treated by a pure frequency-based algorithm as proposed in [6].
Another level of complexity is added by different storage container classes used in the warehouse: As they are of different size, they cannot be randomly exchanged, therefore a standard simulated annealing algorithm cannot be applied. Moreover, the number of different items stored per aisle depends on the composition of container classes in this aisle. This poses additional combinatorial constraints in our implementation.
To sum up, the algorithm proposed below should find out which storage configuration minimises the total retrieval time when items are picked in large batches and several constraints occur on storage containers. We address the following questions: is it best to place all items of one batch in immediate vicinity (see Section 5.3.2 for a definition), even if this results in longer travel times of the picker to reach them? Is

	real	sampled
Cooling factor α	0.97	0.97
Final temperature T_f	0.1	0.1
Initial temperature T_i	10^6	10^6
Sweeps per temperature step	5	5

Table 1: Parameters used in order to perfrom one simulated annealing run. One sweep is defined as a collection of as many moves as there are items in the warehouse.

it most important to store the most used bins in the aisles closest to the entrance of the aisle/warehouse (depending on the chosen routing)? Will the longer pick time on the upper levels disfavor them?

4. Solution approach

We build a simulated annealing algorithm which finds the configuration of warehouse items in a multi-level, multi-container class warehouse, minimising the total retrieval time of batched orders under a given routing. For this, we need to specify (1) how we calculate the retrieval time and (2) which update routine we implement in our SA algorithm to gradually improve retrieval times. The parameters for one simulated annealing run are listed in Table 1.

4.1. Construction of retrieval time

As we consider pre-batched orders, meaning several items collected into the same bin during one route, we have to calculate the retrieval times per ordered bin. The total retrieval time t in the reference time frame is simply the sum over the individual retrieval times $t_{bin,i}$ for each bin (labelled by i) ordered in the reference time frame:

$$t = \sum_{i=1}^{N_{\text{bin}}} t_{\text{bin},i} \tag{4.1}$$

As mentioned above, our algorithm achieves this by optimising the storage assignment while using a heuristic to minimise travel times. The retrieval time per bin t_{bin} is therefore split into the pick time t_p and the travel time t_r. These are calculated independently from one another and the retrieval time per bin then reads

$$t_{\text{bin}} = t_p + t_r. \tag{4.2}$$

4.1.1. Routing heuristic

The *travel time* is the time that a picker needs to physically move to all locations in the warehouse where the goods for one bin are stored. The goal is to reduce the overall travel time for all bins ordered in the reference time frame.

It is easy to see that for a warehouse with only parallel aisles, the s-shaped routing heuristic can be applied both for wide aisles, enforcing a right-hand pick rule, and for narrow aisles (no right-hand rule). However, while unidirectional narrow aisles have to be traveled completely, the routing heuristic in wide aisles would be to pick with the right hand until the last item in the batch for the right part of the aisle is reached, and then to turn around and pick from the "left" part of the aisle, exiting at the same point where the aisle was entered.

We use a routing heuristic optimised for wide aisles. This formula can readily be adapted to an s-shaped routing in unidirectional narrow aisles, for example by a change of indexing of the input data.

Dividing every aisle into subsections, the time needed to travel through one wide aisle while applying a right-hand pick rule is given by the travel time per subsection τ_s multiplied by the distance d of the subsection farthest away from the entrance of the aisle (as this is the point when the picker turns around and starts picking from the other side of the aisle)[1]. In case of narrow aisles, the time needed to travel through one aisle is constant, namely just the number of subsections per aisle multiplied τ_s. In other words, for narrow aisles, d is a constant and not a variable. In formula, for wide aisles the travel time t_r for one bin is calculated as

$$t_r = N_{\text{aisle}} \cdot \tau_{\text{aisle}} + \sum_{a \in \text{aisles}} d_a \cdot \tau_s \tag{4.3}$$

where N_{aisle} is the number of aisles that have to be entered to collect all items in this batch, τ_{aisle} the time needed to change from one aisle to the next and τ_s the time it takes to move one subsection within one aisle.

In case of a narrow aisle, the picker will travel the whole length of the aisle and move to the next aisle at its end (s-shaped routing). Therefore d disappears as a parameter, we can simplify the travel time for one aisle as $\tau_{Ns} = N_s \tau_s$, and formula (4.3) simplifies to $t_r = N_{aisle}\tau_{aisle} + \tau_{Ns}$.

[1] We call d the distance as it can be calculated by any norm which the user considers adapted. For wide aisles, the easiest notion of distance is twice the maximum norm, so that $d = 2\max\{x_j - x\}$, where x is the entrance point of the aisle and x_j the j-th item to be picked in this aisle.

4.1.2. Multi-level picking

The literature often distinguishes between low-level and high-level picking. In low-level picking systems the picker can directly collect the items from the storage racks, while *high-level picking* or "*man-aboard* order-picking" indicates the use of a lifting order-pick truck or crane, see [15] for a detailed exposition. We design our algorithm in a way that the pick times depend on the level where the item is stored. In the simplest case, when only a distinction between low-level and high-level picking is made, we therefore work with two picking times, namely τ_l for the lower level(s) and τ_u for "upper level" picking. The pick time per bin can then be calculated as

$$t_p = N_l \cdot \tau_l + N_u \cdot \tau_u + \Theta(N_u) \cdot \tau_{\text{lift}} \tag{4.4}$$

where N_l is the number of items located in lower levels and N_u the number of items located in the upper levels, respectively. The last term in equation (4.4) adds the time τ_{lift} needed to fetch or adjust the lifting device to the upper level(s). In the simplest situation of only one level change, $\Theta(N_u)$ function returns one if there are any elements to retrieve from the upper levels and zero if the picker has no need to visit the upper levels.

Formula (4.4) is only the special case of the general multi-level picking time formula

$$\tilde{t}_p \;=\; (L-1) \cdot \tau_{\text{lift}} \sum_{j=1}^{L} N_j \cdot \tau_j \tag{4.5}$$

where L is the number of level changes to be made by the picker and τ_j the picking time for an item stored in level j.

By specifying different picking times for different levels or other special situations, the proposed algorithm can be adapted to combined low- and high-level picking, and by adjusting τ_{lift}, τ_{aisle} and τ_s, the algorithm applies also to differently shaped warehouses and variable routing heuristics.

4.2. Construction of moves

As explained in Section 2.3, the cost-function which our simulated annealing algorithm has to minimise is the total retrieval time constructed in (4.1). It is of key importance for the performance of the algorithm to choose the correct type of *moves* to reduce the total retrieval time efficiently.

The specific design of our test warehouse adds several restrictions to admissible moves: the presence of different-size storage containers in the warehouse translate into combinatorial compatibility conditions, i.e. is has to be ensured that the algorithm does not exchange two storage containers of different size.

The crucial observation is that the admissible combinations of container classes form subsections in the aisle. Therefore, in the presented algorithm, we implement two update routines: one routine exchanges two random boxes from the same size category and the other routine swaps whole subsections in two randomly chosen levels and aisles.
With the first move, it can be tested if clustering strongly correlated items is favorable not only to minimise the individual item picking time for one batch, but also globally; The second routine both ensures that a solution found is admissible and it accelerates the optimisation by searching for a more adequate location of a group of already clustered items.

5. Validation and results

5.1. Case description

We test our algorithm with real data from the production site of a medium-size European company offering highly customizable products with a long lifespan. When ordering a product, the customer chooses from a large number of possible options for the product of his choice, which are assembled at the production site.
The individual parts for the product are prefabricated by subcontractors, shipped to the company and stored in the warehouse until needed, there is no just-in-time delivery. Orders to the warehouse arrive as a batch, which is picked in a single journey. The picker stores the items in the batch on a bin, which includes small trays for small pieces and a dedicated space for heavy items, so there is no issue of considering heavy or delicate items when deciding the routing strategy. When all items are collected, the picker delivers them to certain input points along the assembly line.
The content of one batched order varies not only according to the output delivery point, which is the input point of a specific step in the assembly line, but also highly depends on the end-configuration of the product chosen by the customer. In other words, customised product options lead to complex correlations between the items stored in the warehouse. Positively correlated items are more likely to be found in one batched order arriving at the warehouse.

5.1.1. Sample warehouse design used in the algorithm

Our algorithm was designed to be general enough to cope with several features appearing in the warehouse of the abovementioned company, which are variable aisle length, multiple storage levels, and different container types with combinatorial restrictions due to their size.

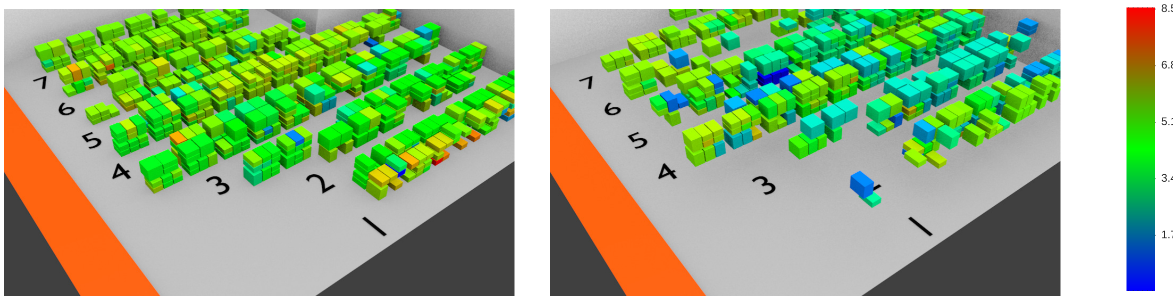

Figure 1: Initial configuration of the warehouse, divided into the different levels: left: lower levels, right: upper levels. The entrance is on the left of aisle 1, the exit area is the orange area on the left. Each box represents a designated storage space for one item. The colours indicate the total picking rate of an item on a logarithmic scale. Red items are much more frequently picked than blue items. The numbers indicate pathways within the aisles. Not every subsection is completely filled due to real world constraints.

As visualised in Figure 1, the sample warehouse of the manufacturer has 7 aisles of variable length (due to the constraints of the production site). The entrance and exit area is located at the left of each aisle. Each aisle is 4 levels high and is divided into (at most) 20 subsections. Individual items are stored in containers of four sizes: Containers have either large or regular sized bottom and are either of single or double height. Each subsection of an aisle is long enough to hold either two containers of large bottom size or three regular containers. Two single-height containers can be stacked on top of each other, therefore, up to six different items can be stored at each level of a subsection.

A total amount of 1268 different components are stored in the warehouse of the manufacturer whose data we used. As can be seen from Figure 1, some storage locations are missing, this is due to specific constraints of the manufacturer's warehouse. In addition, note that some of the seemingly empty storage locations in the less frequented "cold" area of the warehouse are partially used as surplus and refill storage. However, they are not visible in Figure 1 as only the primary pick location of the 1268 individual items is considered in the optimisation. This is not a simplification, as only this primary pick location is mentioned in the order sheet given to the picker. An order batch contains between one and 150 items each. As mentioned, the bin has designated storage options for different items. For example, small items go on the trays, so that no predefined sequence of picking is needed. The delivery locations are designated spots at the edge of the assembly line (orange area of Figure 1). As the retrieval time for one order batch is by orders of magnitude longer than the travelling

time to the delivery points along the assembly line, the influence of those delivery points on the warehouse storage locations can be neglected.
In addition to the sample warehouse given by our industrial partner, we created a virtual warehouse to validate our results. This warehouse contains 240 subsections with 839 items randomly chosen from the original warehouse and distributed in to 6 aisles with one lower and one upper level each. The items are randomly stored. This virtual warehouse as well as the corresponding pre-batched orders and box categories are included in the supplementary material.

5.1.2. Routing heuristics in the company

Our algorithm is designed to adapt to several routing situations. The routing heuristics used in our sample warehouse is as follows: pickers start with picking from the lower two levels of a wide aisle. They follow a right-hand rule, which means that they pick only on their right while travelling along the aisle. Once the last item to their right is reached, they turn around and pick the other side of the aisles until they arrive back to the entrance of this aisle and change to the next aisle. After completing the routing in the lower levels, a lifting order-pick truck is fetched and the picker starts picking the upper levels. Using this routing heuristics, the company arranged their storage allocation (roughly) based on the picking frequency, with some experience-based modifications. This frequency-experience-based heuristics already reduced the total retrieval time significantly, and our algorithm displays this fact correctly, as we show in the beginning of Section 5.3.

5.1.3. Individual pick times

Note that the time required for each pick (denoted by τ_j in formula (4.5)) changes depending on the level from which items are picked. To simplify the analysis our sample warehouse, we consider the simple situation of only two different picking times, τ_l for the lower levels, and τ_u for the upper levels, see formula (4.4). The time required for each pick in the upper levels is significantly longer than for the lower levels, as more careful steering is needed and the picker is less mobile. The choice of pick times and the time for aisle change used in the experiments of our algorithm are listed below. Please recall equations (4.3) and (4.4) for the definition.
It is worth mentioning that, qualitatively, the solution does not depend on the exact times as long as they are in reasonable proportions to each other. Note that all times listed in the below table are based on the experience of our industrial partner, they have been determined by measurement on-site, but might vary slightly depending on individual picker skills.

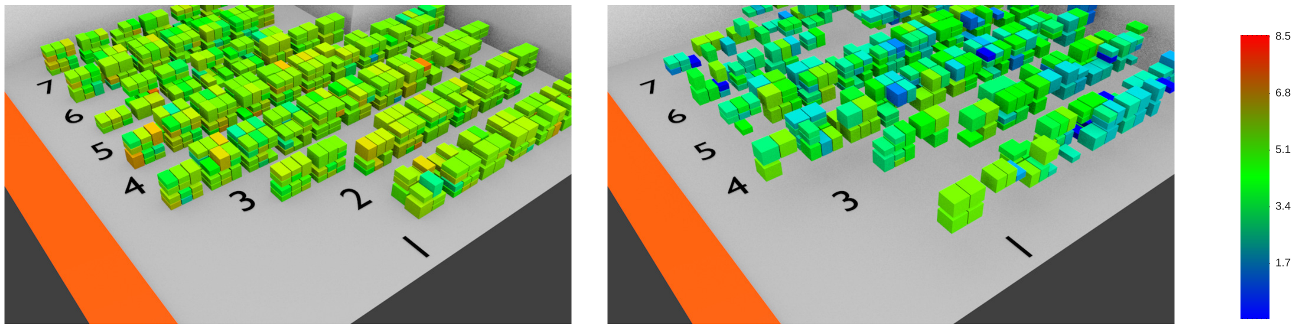

Figure 2: Optimised warehouse design divided into the different levels: left: lower levels, right: upper levels. The colours indicate the logarithm of the total picking rate of an item. Red items are much more frequently picked than blue items.

Variable	Time in seconds
τ_{aisle}	30
τ_s	2 per subsection
τ_l	15
τ_u	30
τ_{lift}	120

5.2. Validation of method

The experiments on the "real" sample warehouse were carried out with 4192 pre-batched orders, representing the products assembled in the reference time frame. For the virtual warehouse, we randomly chose several orders from these products resulting in 2592 pre-batched bins, to keep the correlations between the items as intact as possible, which is necessary to make the results comparable. All experiments were run single-threaded on Intel(R) Core(TM) i7-4790 CPU running at 3.60GHz and with 8 GB of memory. We used a geometric cooling schedule starting at $T = 10^6$ and stopping when convergence is reached with a cooling constant $\alpha = 0.97$, resulting in about $3.3 \cdot 10^6$ iterations. The total running time for one annealing was about 12 hours.

5.3. Results

(1) Significant reduction of total retrieval time. After near-optimisation of storage locations by our algorithm, the total retrieval time for all 4192 pre-batched orders in the reference time frame was calculated to be approximately 38% lower than for a random item distribution (virtual warehouse) and 21% lower compared to the total

retrieval time with the initial, experience-based storage location configuration used by the manufacturer.
This is a significant improvement, in particular as the initial storage allocation essentially applied frequency-based heuristics similar to [6]: Fixing the routing heuristics, the company's logistics division had already re-arranged the items used more often to the first aisles and those used less were put in the back aisles. Manual modifications to this heuristics were made according to experienced correlations. However, as our results show, the correlations given by large batched orders are too complex to be solved well in a heuristical way.

(2) Stability of configuration under order changes. The results in paragraph (1) considered a near-opimisation of the warehouse configuration based on historical orders. An interesting question is to see how this configuration performs under changes in customer wishes, i.e. under different orders. If the sensitivity under order changes is too high, the manufacturer would need to re-arrange its warehouse configuration too often, leading to a decrease in productivity.
To investigate this question, we sampled ten shorter sequences of pre-batched orders with replacements from the empirical product orders, keeping the correlations between the items as intact as possible. This new sequences contain about 2300 pre-batched orders each. Note that it is important to preserve the correlations in order to model a realistic situation - to destroy the batch structure means to pick items which do not add up to a real product (e.g. trying to assemble a car without screws).
For these new batches, in comparison to the initial sample warehouse configuration, the near-optimal configuration proposed by our algorithm reduced the total retrieval time by 19.9%.
Rearranging the virtual warehouse with the algorithm, i.e. optimising a random intial item distribution with a different set of batched orders, results in a 40% lower total retrieval time. This improvement reduces to 25% if the storage locations in the virtual warehouse are sorted based on the pick-frequency of each subsection.
From these tests we conclude that near-optimal configuration found by our algorithm seems to be stable under realistic order changes, while the precise reduction of retrieval times might vary slightly.

5.3.1. Detailed analysis - global distribution

Figure 2 shows the optimised warehouse design. Red items have the highest picking frequency, blue items the lowest. In comparison to the starting situation (Figure 1), the lower levels appear to be better stocked. They contain the more frequently picked items, while the upper levels contain mainly items which are less frequently

picked. In fact, the algorithm slightly increased the average picking-frequency on the lower levels, which is to be expected since the higher picking time in the top floors and the need for a lifting device have a significant impact on the optimisation.
The first aisle, which is the closest to the entrance, initially had a lot of very frequently picked items on the first two levels, as this was the preferred storage location in the heuristics used by the company. After optimisation by our algorithm, most of these items were moved, as the aisle is (a) very short and (b) has only one side, it is therefore less efficient to visit from a global optimisation point of view, as taken by our algorithm.

5.3.2. Detailed analysis - batch-induced correlations

As already discussed above, the picking process of large batched orders results in complex correlations between the individual items in the warehouse. The correlations are not necessarily related to the frequency of picks of a single item: some items are very basic and are used for every single product that this company is producing, independently of the product option chosen by the customer, while other items are specific to one product option and appear in exactly one batch if and only if this product option was ordered.
One might conjecture that clustering the items according to their correlation to each other leads to a lower total retrieval time. By clustering we mean that highly correlated items are stored in neighbouring storage containers.
To check if our algorithm does indeed cluster correlated items, we visualise in Figure 3 the changes in correlation between neighbouring items. The visualisation is done via the average jaccard-similarity coefficient of a batch which contains item i and a batch which contains a direct neighbour of item i, which we call j. Roughly speaking, a high jaccard-similarity coefficient means that item i and its neighbour j are positively correlated in the picking process, i.e. a large percentage of batches which contain i also contain j.
The calculation of the jaccard-similarity coefficient goes as follows: Denote $\{B_i\}$ a batch in which i occurs and $\{B_j\}$ a batch in which j occurs. The jaccard-similarity coefficient measures the "similarity" between two finite sample sets $\{B_i\}$ and $\{B_j\}$ and is defined by

$$\mathrm{sim}\left(\{B_i\},\{B_j\}\right) = \frac{|\{B_i\} \bigcap \{B_j\}|}{|\{B_i\} \bigcup \{B_j\}|}. \tag{5.1}$$

Moreover, define the set of neighbours of item i as the set of items j, which are in the same or adjacent subsection of an aisle and their corresponding opposite subsection. We require that j has to be stored in the same level category as i, meaning that if i

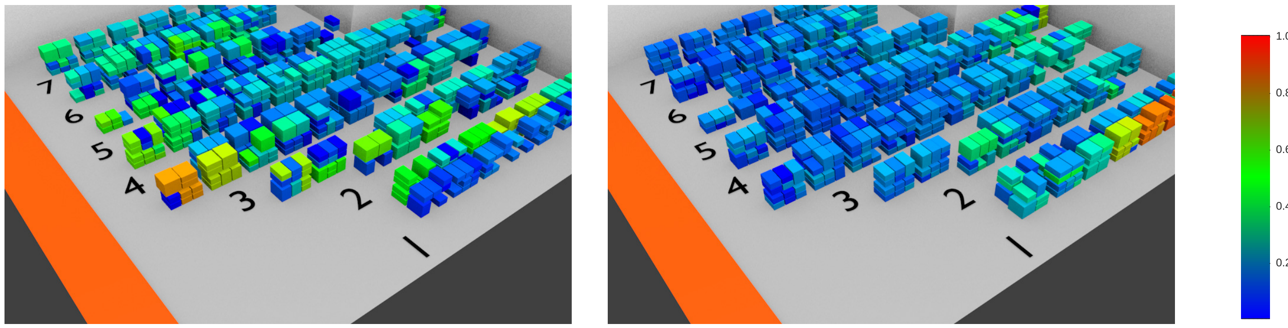

Figure 3: The average jaccard-similarity for each item in the lower levels before (left) and after (right) optimisation

is stored in a lower level, then also j has to be stored in a lower level to qualify as a neighbour.

Figure 3 shows that the heuristic approach taken by the company shows a higher average similarity, i.e. objects which are often picked to the same batch are stored in neighbouring containers. However, the optimised configuration displays a lower overall similarity and shows high similarity only for items in the first aisle, which is a special case as there are very few items, which are overall rarely picked.

Consequently, in contrast to intuition, clustering of correlated items does not necessarily lead to a reduction in retrieval times. This phenomenon can be explained with the relatively low impact of item distance to the total retrieval time of a large batch, in relation to the sum of the picking times of the individual items. This behaviour is also present in the other warehouse levels. Performing a jaccard-similarity analysis in the virtual warehouse shows that the similarity is significantly above average only at the end of the aisles.

We conclude that clustering seems to improve the total retrieval time only if the cost of picking this item is high. This explains also the high similarity of items stored in aisle one: this aisle is shorter and contains only a limited number of items, so it is worth visiting only if several items need to be picked together.

To sum up the analysis, the complex structure of the batches leads to highly non-trivial optimised configurations, which cannot be fully explained by the positive correlations of neighbouring items. Moreover, the near optimal storage configuration does not agree with the configuration given by a frequency-based heuristics or division in “hot areas” and “cold areas” either.

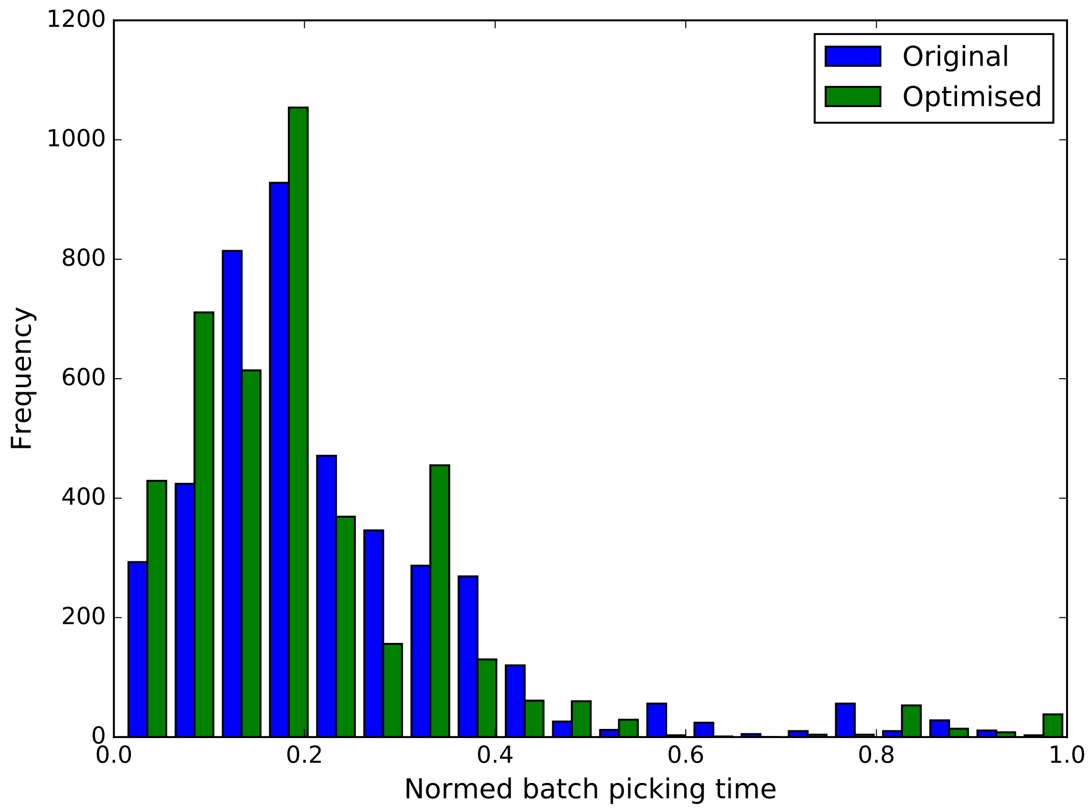

Figure 4: Histogram of individual batch picking times normed to the maximum of the original picking time (blue) before and (green) after optimisation.

5.3.3. Detailed analysis - individual picking times

Another interesting question is whether a near-minimal total retrieval time for a large number of batches might go to the expense of very long retrieval times for a few batches. Or, more generally, if the distribution of the retrieval time per batch changed after the optimisation.

In the last step of our analysis we compare the picking times per individual batch. To compare the distribution, the displayed picking times are normed to the maximum batch picking time in the original configuration. The histogram of individual picking times are shown in Figure 4: Before the optimisation, the distribution of individual picking times is centred around a value of 0.226 with a right handed fat tail. The algorithm is able to shift the distribution slightly to shorter times, the mean decreases to 0.204, but the fat tail cannot be removed. Note that some batches contain more than 100 items, so it makes sense if its retrieval time is still very long.

To conclude, the near-optimal storage assignment obtained by our algorithm does not necessarily lead to a different distribution of picking times per batch or a lower probability of extremely long retrieval times for arbitrary batches. However, retrieval times per batch get shorter on average, leading to the improvements discussed in Section 5.3.

6. Discussion/Conclusion

This paper presents a simulated annealing algorithm to reduce picking times of potentially large sets of batched orders by a optimisation of the storage assignments in a multi-level warehouse. The main novelty of our method is to provide a structured approach for the storage location assignment problem with very general pre-batching constraints and adapted to a multi-level warehouse setting. In particular, the algorithm is able to deal with complex correlated batches and able to find highly non-trivial optimised configurations.

To our knowledge, our article is the first to investigate the impact of large batched orders on storage assignment solutions. Contrary to intuition, we give evidence that, even for batched orders of very large size (e.g. over 100 items in one batch), clustering of correlated items in specific parts of the warehouse does not necessarily lead to a reduction in retrieval times. Indeed, our simulations show that clustering of items is only beneficial if the cost to retrieve the items is relatively high, i.e. when they are located at an unfavorable location (end of an aisle, unfavorable aisle etc).

Our main contribution is a general solution method for the solution of the storage location assignment problem under adaptable routing methods, which is flexible enough to be used in a multi-level warehouse setting with different container sizes.

While the current algorithm was designed for single-block warehouse layouts, extensions to more general warehouse settings can easily be done by adapting formula (4.3). Notably, this algorithm can also be applied to parts of the warehouse without resulting in phantom constraints or other unnatural solutions to this optimisation problem.
We tested our algorithm on real data, optimising the storage assignments in a four-level warehouse of a manufacturer with 4192 pre-batched orders of 1-150 items per batch. The storage assignment suggested by the algorithm reduces the total retrieval time by 21% compared to heuristics based on the frequency of picking for individual pieces. Assuming a random item distribution in the warehouse, this simulated annealing algorithm reduces the overall picking time approximately by 38%. These savings are achieved without changing the routing heuristics and without splitting large pre-batched orders. The simultaneous delivery of all items, even of a very large batch, is crucial to maintaining efficiency of the assembly line. Note that the algorithm is very fast, as all parts of the algorithm are designed and optimised for multi-level warehouses. No black-box packages are used.

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