Multi-Scale Analysis by Γ-convergence of a Shell-Membrane Transition

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Abstract

We study the asymptotic behavior of functionals associated to the energy of a thin nonlinear elastic spherical shell in the limit of vanishing thickness (proportional to a small parameter) ε and under the assumption of radial deformations. The functionals are characterized by the presence of a nonlocal potential term and defined on suitable weighted functional spaces. The transition shell-membrane is studied at three relevant different scales. For each of them we give a compactness result and compute the Γ-limit. In particular, we show that if the energies on a sequence of configurations scale as $\varepsilon^{3/2}$ then the limit configuration describes a (locally) finite number of transitions between the undeformed and the everted configurations of the shell. We also highlight a kind of 'Gibbs' phenomenon' by showing that non-trivial optimal sequences restricted between the undeformed and the everted configurations must have energy scaling at least as $\varepsilon^{4/3}$.