A CRITERION FOR PURE UNRECTIFIABILITY OF SETS (VIA UNIVERSAL VECTOR BUNDLE)

SILVANO DELLADIO

Abstract. Let m, n be positive integers such that $m < n$ and let $G(n,m)$ be the Grassmann manifold of all m- dimensional subspaces of $\mathbb{R}^n$. For $V \in G(n,m)$ let π_V denote the orthogonal projection from $\mathbb{R}^n$ onto V. The following characterization of purely unrectifiable sets holds. Let A be a $\mathcal{H}^m$-measurable subset of $\mathbb{R}^n$ with $\mathcal{H}^m(A) < \infty$. Then A is purely m-unrectifiable if and only if there exists a null subset Z of the universal bundle $\{(V,v) \,|\, V \in G(n,m),\, v \in V\}$ such that, for all $P \in A$, one has $\mathcal{H}^{m(n-m)}(\{V \in G(n,m) \,|\, (V, \pi_V(P)) \in Z\}) > 0$. One can replace "for all $P \in A$" by "for $\mathcal{H}^m$-a.e. $P \in A$".

1991 *Mathematics Subject Classification.* Primary 28A75, 28A78, 49Q15; Secondary 53A05 .
Key words and phrases. Purely unrectifiable sets, Rectifiable sets, Geometric measure theory.