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Preface

In the last 30 years several problems have been examined in the framework of the study of certain composite materials having the particular feature that they can be described by means of minimizing configurations of energies not necessarily finite on all the “smooth” admissible ones.

Problems involving energies with these features appeared, for example, in the study of elastic-plastic torsion theory, of electrostatic screening, and of the modelling of some rubber-like nonlinear elastomers, and have been generally approached by means of ad hoc, or particular mathematical techniques.

The aim of the present volume is to propose a systematic and unifying mathematical framework, within the calculus of variations, for the treatment of problems of this nature, at least in the stationary case.

From this point of view, the fundamental notion that appears to play a central role is the one of *unbounded functional*. These functionals take nonnegative extended real values, and represent the energies under consideration. They depend, in a classical manner, essentially on two variables: one of set-type nature in which the functional enjoys measure theoretic properties, and one of scalar configuration-type nature in which it enjoys convexity and lower semicontinuity properties. On the other side, the above energies behave also in a “non-classical” way. They turn out to take finite values only on those configurations that are subject to pointwise constraints on the strains, hence not depending on the regularity of the configurations themselves.

The analysis of this notion requires the reconsideration of well-established concepts and techniques. Therefore the book naturally divides into two parts.

In the first part (Chapters 1 to 5), we aim to allow as much as possible a self-contained reading of the volume. The main notions of convex analysis are recalled, together with those of measure theory, and of theory of variational convergences. Then we introduce some function spaces usually considered in calculus of variations, where we study some lower semicontinuity and minimization problems for energy functionals. Such notions are also adapted to the new setting by means of the necessary changes and the required extensions.

At the end of the first part, Chapter 6 plays the role of a hinge chapter. It begins with a brief survey on some aspects of the theory of the standard functionals of the calculus of variations such as unique extension properties, representation as integrals of the calculus of variations, relaxation theory, and homogenization processes. Then, the mathematical aspects of some physical models, which suggest the notion of unbounded functional, are briefly explained.

By unique extension properties, we mean those types of problems in which one tries to extend a function defined in a set to a wider one by preserving some of its characteristic features, and gaining uniqueness of the extension.

The representation as integrals of the calculus of variations problems refers to the identification of sufficient conditions (possibly also necessary) on an abstract functional F implying its description as

$$F(\Omega, u) = \int_{\Omega} f(x, \nabla u)dx,$$

where Ω is the set-type variable and u the configuration-type one.

Given a function F defined in a topological space, relaxation problems deal with the study of representation formulas for the description of the relaxed function of F, namely of the greatest lower semicontinuous function less than or equal to F, having in mind the qualitative property according to which the greatest lower bound of a function agrees with the minimum of its relaxed function.

By homogenization problems we mean those in which one tries to simulate the behaviour of composite materials finely grained in a "regular" way (somehow comparable to a periodic distribution of two or more components) by means of a homogeneous one, and vice-versa. In this book, we restrict ourselves to the cases where such simulation can occur in the sense that the minimum energy of the homogeneous material turns out to be close, for every admissible external force, to the one of the composite materials, as much as the graining is fine.

In the physical models inspiring unbounded functionals, the energies involved have an integral form on "regular" configurations, but the energy densities f are unbounded.

Thus, in the second part of the volume (Chapters 7 to 13), which is the most original one, a tentative theory of unbounded functionals is developed according to the scheme proposed in Chapter 6, having in mind the described models and focusing mainly on homogenization. This is done, at least in the case of unique extension, integral representation and relaxation, for "translation invariant" functionals, i.e. functions that don't change their values when both the set-type variable and the configuration-type one undergo translations.

Finally, in Chapter 14, the homogenization results obtained are exploited to provide some explicit descriptions of the homogenized materials relative to the unbounded energies proposed in Chapter 6.

In our opinion, the theory developed in such a way allows to obtain deeper results than the already known ones, and to address interesting new problems, including ones in applied mathematics.

In memory of Ennio De Giorgi and Jacques-Louis Lions, we would like to point out that several ideas contained in this book originated from their scientific visions and mathematical concepts.

We are also indebted to Haïm Brezis for his warm encouragements in the preparation of the book and for some deep discussions, and to Sergio Spagnolo for his friendly mathematical teachings.

Finally, we want to remark that the research activities on composite materials can be considered as a common effort, to which a lot of mathematicians contribute with different competencies. So we are also indebted to many colleagues for several comments and discussions.

The book contains both published and new results. It is mainly aimed at graduate students and researchers in mathematics, but we hope that it may be useful to engineers and continuum physicists.

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