ERRATA AND ADDITIONS TO MY BOOK "LECTURE NOTES ON MEAN CURVATURE FLOW"

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Thanks to Giovanni Bellettini, Or Hershkovits, Matteo Novaga for finding several mistakes and inaccuracies in my book.
This errata does not cover the English mistakes but only the mathematical arguments and typos. Comments/suggestions and highlighting of other mistakes are strongly welcome.

28^{17} : Replace Theorem 2.2.1 and the first paragraph of its proof with the following:

Theorem 2.2.1 (Comparison Principle for Mean Curvature Flow). *Let* $\varphi : M_1 \times [0,T) \to \mathrm{R}^{n+1}$ *and* $\psi : M_2 \times [0,T) \to \mathrm{R}^{n+1}$ *be two hypersurfaces moving by mean curvature, with* M_1 *compact. Then the distance between them is nondecreasing in time.*

Proof. The distance between the two hypersurfaces $\varphi_t : M_1 \to \mathrm{R}^{n+1}$ and $\psi_t : M_2 \to \mathrm{R}^{n+1}$ at time t is given by $d^{\varphi}_{\psi}(t) = \inf_{p\in M_1, q\in M_2} |\varphi(p,t) - \psi(q,t)|$. As the mean curvature is uniformly bounded in space and locally in time for both hypersurfaces, this function is locally Lipschitz, hence differentiable almost everywhere, we assume in the following that t is a differentiability point.

42_1 : Replace $|\mathrm{A}|^2 = \mathrm{S}_1^2 + 2\mathrm{S}_2$ with $|\mathrm{A}|^2 = \mathrm{S}_1^2 - 2\mathrm{S}_2$

63^8 : Replace $d_i(p_i.q_i) = \varepsilon$ with $\widetilde{d}_i(p_i, q_i) = \varepsilon$

63_{12} : Replace $\widetilde{d}_{t_i}(p_i.q_i) > \varepsilon$ with $\widetilde{d}_i(p_i, q_i) > \varepsilon$

71^6 – end of the page : Replace this halfpage with the following:

For every $\varepsilon > 0$, as $\Theta(p_i) \geq 1$ we have definitely

$$\begin{aligned}
\varepsilon \geq \theta(p_i,t_i) - 1 \geq \theta(p_i,t_i) - \Theta(p_i) \\
&= \int_M \frac{e^{-\frac{|x-\widehat{p}_i|^2}{4(T-t_i)}}}{[4\pi(T-t_i)]^{n/2}}\, d\mu_{t_i} - \Theta(p_i) \\
&= \frac{1}{(2\pi)^{n/2}} \int_M e^{-\frac{|y|^2}{2}}\, d\widetilde{\mu}_{i,s_i} - \Theta(p_i) \\
&= \frac{1}{(2\pi)^{n/2}} \int_{s_i}^{+\infty} \int_M e^{-\frac{|y|^2}{2}} \left| \widetilde{\mathrm{H}} + \langle y \,|\, \widetilde{\nu} \rangle \right|^2 d\widetilde{\mu}_{i,s}\, ds\,.
\end{aligned}$$

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Hence, since by the uniform curvature estimates of Proposition 3.2.9, see computation (3.2.9), we have,

$$\left|\frac{d}{ds}\int_M e^{-\frac{|y|^2}{2}}\left|\widetilde{\mathrm{H}}+\langle y\,|\,\widetilde{\nu}\rangle\right|^2\,d\widetilde{\mu}_s\right|\leq C$$

where $C=C(\mathrm{Area}(\varphi_0),T)$ is a positive constant independent of s, we get

$$\begin{aligned}\varepsilon&\geq\frac{1}{(2\pi)^{n/2}}\int_{s_i}^{+\infty}\int_M e^{-\frac{|y|^2}{2}}\left|\widetilde{\mathrm{H}}+\langle y\,|\,\widetilde{\nu}\rangle\right|^2\,d\widetilde{\mu}_{i,s}\,ds\\&\geq\frac{1}{(2\pi)^{n/2}}\int_{s_i}^{s_i+\frac{1}{C}\int_M e^{-\frac{|y|^2}{2}}\left|\widetilde{\mathrm{H}}+\langle y\,|\,\widetilde{\nu}\rangle\right|^2\,d\widetilde{\mu}_{i,s_i}}\Bigl(\int_M e^{-\frac{|y|^2}{2}}\left|\widetilde{\mathrm{H}}+\langle y\,|\,\widetilde{\nu}\rangle\right|^2\,d\widetilde{\mu}_{i,s_i}-C(s-s_i)\Bigr)\,ds\\&=\frac{1}{(2\pi)^{n/2}}\frac{1}{2C}\Bigl(\int_M e^{-\frac{|y|^2}{2}}\left|\widetilde{\mathrm{H}}+\langle y\,|\,\widetilde{\nu}\rangle\right|^2\,d\widetilde{\mu}_{i,s_i}\Bigr)^2.\end{aligned}$$

If now we proceed like in Proposition 3.2.10 and we extract from the sequence of hypersurfaces ψ_i a locally smoothly converging subsequence (up to reparametrization) to some limit hypersurface $\widetilde{M}_\infty$, by Lemma 3.2.7 we have

$$\varepsilon\geq\frac{1}{(2\pi)^{n/2}}\frac{1}{2C}\Bigl(\int_{\widetilde{M}_\infty}e^{-\frac{|y|^2}{2}}\left|\widetilde{\mathrm{H}}+\langle y\,|\,\widetilde{\nu}\rangle\right|^2\,d\widetilde{\mathcal{H}}^n\Bigr)^2,$$

for every $\varepsilon>0$, hence $\widetilde{M}_\infty$ satisfies $\widetilde{\mathrm{H}}+\langle y\,|\,\widetilde{\nu}\rangle=0$.

72^5 : Replace A^i and $|\mathrm{A}^i(p)|$ respectively with $\widetilde{\mathrm{A}}^i$ and $|\widetilde{\mathrm{A}}^i(p)|$

73_5 : Replace the formula with the following one:

$$\sigma(0)-\Sigma=\frac{1}{(2\pi)^{n/2}}\int_{-\frac{1}{2}\log T}^{+\infty}\int_M e^{-\frac{|y|^2}{2}}\left|\widetilde{\mathrm{H}}+\langle y\,|\,\widetilde{\nu}\rangle\right|^2\,d\widetilde{\mu}_s\,ds<+\infty\,,$$

73_3 : Replace the formula with the following one:

$$\frac{1}{(2\pi)^{n/2}}\int_{\widetilde{M}_\infty}e^{-\frac{|y|^2}{2}}\,d\widetilde{\mathcal{H}}^n=\Sigma\geq 1\,.$$

81^1 : Replace $h_{ij}v^j_k(s)=$ with $h_{ij}v^j_k(s)=0$

81^{13} : Replace "By Sard's theorem, there exists a vector $y\in S$ such that" with "Since the orthogonal projection map $\pi:M\to S$ is then a submersion, for every vector $y\in S$ we have that"

96^{17} – end of Proof of Proposition 4.2.3 : Replace with the following:

identically zero and also A_∞ would be identically zero (by Proposition 2.4.1 and the pinching estimates in Corollary 2.4.3, which are invariant by rescaling and pass to the limit), in contradiction with the fact that the limit flow is nonflat.

96_{10} : Replace $\mathrm{H}_k(q_k, s) = \mathrm{H}(q_k, s/Q_k + t_k)/Q^k \to \mathrm{H}_\infty(p, t) > 0$ with $\mathrm{H}_k(q_k, s) = \mathrm{H}(q_k, s/Q_k^2 + t_k)/Q_k \to \mathrm{H}_\infty(p, t) > 0$

96_9 : Replace $\mathrm{H}(q_k, s/Q_k + t_k) \to +\infty$ with $\mathrm{H}(q_k, s/Q_k^2 + t_k) \to +\infty$

96^5 : Replace the formula with the following one:

$$\frac{\lambda_k^{\min}(q_k, s)}{\mathrm{H}_k(q_k, s)} = \frac{\lambda^{\min}(q_k, s/Q_k^2 + t_k)}{\mathrm{H}(q_k, s/Q_k^2 + t_k)} \geq -\eta - \frac{C}{\mathrm{H}(q_k, s/Q_k^2 + t_k)}$$

117 : Replace the first paragraph in Subsection 5.2.5 with the following:
About the evolution of a generic compact, smooth, n–dimensional, initial hypersurface we can only say that if it is initially embedded, it stays embedded, when it develops a type I singularity we can produce a (possibly flat) homothetically shrinking hypersurface as a blow up limit, nonflat in the embedded case. If the singularity is of type II then Hamilton's procedure gives a blow up limit which is an eternal flow with bounded curvature, such that $|\mathrm{A}|$ achieves its absolute maximum at some point in space and time.

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