Online workshop in Geometric Analysis

15/12/2020 - 17/12/2020

Book of abstracts

Limits of Riemannian manifolds with a bound on the Kato constant

Ilaria Mondello (Laboratoire d'Analyse et Mathématiques Appliquées, Université de Paris-Est Créteil)

The structure of non-collapsed limits of manifolds with a uniform lower bound on the Ricci curvature is now well known, thanks to the work of many mathematicians, Anderson, Cheeger, Colding, Tian, Naber, Jiang. In this talk, we discuss the study of limits of manifolds with a weaker integral condition on the Ricci curvature. After introducing this condition and the definition of Kato limits, we show that non-collapsed Kato limits carry a stratification similar to the one of Ricci limits and RCD spaces. This is a joint work with Gilles Carron (Université de Nantes) and David Tewodrose (Université Libre de Bruxelles)

The sharp isocapacitary inequality: the case of p-capacity

Ekaterina Mukoseeva (University of Helsinki)

It is well-known that the smallest p-capacity among sets of a given volume is achieved solely by balls. A natural question to ask is whether the balls are stable minimizers. That is, supposing a set has almost the p-capacity of a ball with the same volume, is it quantitatively close to it? The first result to our knowledge was given by Fusco, Maggi, and Pratelli in 2009. However, the estimate they provide is not sharp. In this talk we discuss the sharp quantitative stability of the balls for p-capacity. The proof combines the Selection Principle, introduced by Cicalese and Leonardi in 2012, with the regularity theory for free boundary problems established by Danielli and Petrosyan in 2005. This is a generalization of a joint work with Guido De Philippis and Michele Marini, where we got the sharp isocapacitary inequality for $p = 2$.

Codimension two area and Yang-Mills-Higgs

Alessandro Pigati (Courant Institute of Mathematical Sciences)

While the calculus of variations of the area of hypersurfaces is becoming more and more understood, with recent developments in the construction of unstable critical points, little is known for higher codimension. In this talk we present an approach to the codimension two, based on the Yang-Mills-Higgs energy for Hermitian line bundles, in the self-dual case. This functional, which is well known in gauge theory, turns out to be a good relaxation of the (codimension two) area functional, in a similar way as the Allen-Cahn functional in codimension one: in joint work with Daniel Stern we show that, if one uses scalings which preserve the self-duality, the energy of critical points concentrates along a minimal variety. We also discuss the corresponding Gamma-convergence theory, which is joint work with Daniel Stern and Davide Parise.

On the smooth convergence of geometric flows

Marco Pozzetta (Università di Pisa)

In this talk we consider extrinsic geometric energies, i.e., functionals $\mathcal{F}$ defined on smooth immersions $\varphi : M \hookrightarrow \mathbb{R}^n$ of a given manifold M depending on extrinsic geometric quantities (such as the second fundamental form, the mean curvature, derivatives of those, ...) such that $\mathcal{F}$ is invariant w.r.t. diffeomorphisms of M. Then an extrinsic geometric flow is a smooth family of immersions $\varphi_t : M \hookrightarrow \mathbb{R}^n$, for $t \in [0, T)$, such that a given extrinsic geometric energy $\mathcal{F}$ evaluated on φ_t decreases in time by some assigned evolution equation.

If such a geometric flow φ_t is defined for any positive time, i.e., $T = +\infty$, we are interested in the existence of a limit smooth immersion φ_∞ of φ_t as $t \to +\infty$. In such a case, we say that the flow *smoothly converges*. In this talk we discuss a strategy for promoting the *sub-convergence* of such a flow, that is the existence of a limit $\lim_n \varphi_{t_n}$ up to isometries of $\mathbb{R}^n$ and on suitable diverging sequences t_n, into the smooth convergence of the flow. In the discussion we analyze the explicit example of the L^2-gradient flow of the energy

$$\mathcal{F}_m(\varphi) := \int_M 1 + |\nabla^m \nu|^2 \, \mathrm{d}\mu,$$

where $\varphi : M^n \hookrightarrow \mathbb{R}^{n+1}$ is an immersed closed hypersurface, ν is a (locally defined) unit normal vector, ∇ the covariant derivative, μ the volume measure, and $m > n/2$ is integer. The strategy of the proof is based on the application of a Łojasiewicz–Simon gradient inequality. We discuss how same argument can be applied also for deducing the smooth convergence of the L^2-gradient flow of the elastic energy of closed curves in $\mathbb{R}^n$, and of the $(L^p, L^{p'})$-gradient flow of the p-elastic energy of closed curves immersed in arbitrary Riemannian manifolds $(\overline{M}, \overline{g})$ for $p \in [2, +\infty)$.

Part of the results discussed in this talk are obtained in collaboration with Carlo Mantegazza (Università Federico II di Napoli & Scuola Superiore Meridionale).

Serrin's type problems in Riemannian manifolds

Alberto Roncoroni (Universidad de Granada)

The talk deals with overdetermined elliptic boundary value problems for bounded domains in Riemannian manifolds. Starting from the well-known Serrin's rigidity result we will consider its counterpart in space forms (the hyperbolic space and the hemisphere). Then we will consider Serrin's type problems in more general Riemannian manifolds. In particular we will show rigidity results for overdetermined problems in the so-called warped product manifolds. This is based on a joint work with Alberto Farina.

The volume-preserving Willmore flow

Fabian Rupp (Ulm University)

For an immersed surface in $\mathbb{R}^3$, we study the L^2-gradient flow of the Willmore energy subject to a volume constraint. Despite its nonlocal nature, we show that the flow can be controlled by the concentration of curvature. This is used to prove a lower bound on the existence time, which enables us to construct a blow-up as it was done by Kuwert and Schätzle for the Willmore flow. With the help of a constrained Lojasiewicz–Simon inequality and a classification result for the concentration limit we show convergence to a round sphere if the initial energy is below 8π.