UNIVERSITY OF WARSAW

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Banach Center
INSTITUTE OF MATHEMATICS PAS

VII Spring School of Analysis in memory of Aleksander Pełczyński

Differentiation and Geometry

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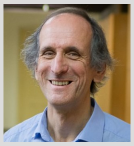
Lecturers:

Giovanni Alberti (Università di Pisa)

Rectifiable decompositions of measures and applications.

Jan Malý (Univerzita Karlova)



Weakly differentiable mappings.

Jean Van Schaftingen (UCLouvain)

Endpoint Sobolev estimates for vector fields and canceling differential operators.

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Abstracts

Giovanni Alberti - Rectifiable decompositions of measures and applications.

In recent years it has been understood that integral decomposition of (positive, finite) measures in terms of rectifiable measures (that is, measures that are absolutely continuous with respect to length measures on rectifiable curves) play a crucial role in several problems in Real Analysis and Geometric Measure Theory. In these lectures I will describe a few examples. One is the extension to Rademacher theorem to measures that are singular with respect to the Lebesgue measure: the classical version of this theorem states that every Lipschitz function is differentiable almost everywhere with respect to the Lebesgue measure, and one may ask how this statement should be modified when the Lebesgue measure is replaced by a singular measure, and it turns out that the differentiability properties of Lipschitz functions are exactly described by the decompositions of the measure in terms rectifiable measures. Another relevant example is the (non) closability of the gradient operator with respect to singular measures. Given a measure μ we can describe the tangent directions of all rectifiable decompositions of μ using a suitable distribution of planes (of variable dimension) called "decomposability bundle" of μ. All the problems mentioned above lead to the following natural question: is it true that every measures μ on $\mathbb{R}^d$ for which the decomposability bundle has dimension d (at μ-a.e. point) must be absolutely continuous with respect to the Lebesgue measure? It has been recently proved that this question has positive answer for every d, and I will describe the proof for $d = 2$, and, if time permits, give an idea of the proof for larger d. The results presented in these lecture are based on the work of many authors, including myself, David Bate, Marianna Csornyei, Guido De Philippis, Peter Jones, Andrea Marchese, Andras Mathe, David Preiss, Filip Rindler and Andrea Schioppa.

Jan Malý - Weakly differentiable mappings.

The study of nonlinear classes of weakly differentiable (Sobolev or BV) mappings is motivated by possible applications in continuum mechanics. They appear as classes of admissible deformations. The theory of mapings of finite distortion developed parallelly from conformal geometry as a far reaching generalization. We survey recent progress in the field of weakly differentiable mappings and then pass to latest results concerning the relation between the regularity of a homeomorphism and the regularity of its inversion in terms of norms of weak gradients and their minors. The case when the gradient or its minor is a measure is also considered. Best results are available in low dimensions. In the planar case it is possible to obtain rectifiability of the graph of a BV homeomorphism. Also in the three-dimensional case, the surface integration of differential forms over the graph of the mapping is a powerful tool. The calculus of jacobians and adjugates in the sense of distributions is relevant in the search for sharp results.

Jean Van Schaftingen - Endpoint Sobolev estimates for vector fields and canceling differential operators

The starting question is how to generalize the endpoint p=1 Sobolev embedding of Gagliardo and Nirenberg to classes of homogeneous vector differential operators. Outside of the endpoint cases, the classical Calderón–Zygmund estimates show that the ellipticity is necessary and sufficient to control all the derivatives of the vector field. In the endpoint case, Ornstein have showed that there is no nontrivial estimate on same-order derivatives. Moreover, the ellipticity is necessary for endpoint Sobolev estimates. Such estimates where proved first for the deformation operator (Korn–Sobolev inequality by M.J. Strauss) and for the Hodge complex (Bourgain and Brezis). The class of operators for which estimates holds can be characterized by a canceling condition. The estimates rely on a duality estimate for L^1 vector fields satisfying some conditions on the derivatives, combined with classical algebraic and harmonic analysis techniques. This characterization unifies classes of known inequalities and extends to the case of Hardy inequalities.

Conference fee : 130 EUR (meals and hotel included)