Alessio Figalli, Fields medallist 2018
Pisa, January 14-17, 2019
Preliminary schedule

	Monday 14, SNS (Sala Azzurra and Aula Bianchi)
9.30–10.30	**Registration (Aula Bianchi) and welcome coffee (Corridoio II piano)**
10.30–11.30	**Institutional greetings and videos**
11.30–12.00	**Guido De Philippis.** *My work with Alessio on Sobolev regularity of the Monge-Ampère equations.*
12.00–12.30	**Aldo Pratelli.** *The quantitative isoperimetric inequality via mass transportation: a work with Francesco Maggi and Alessio.*
	Lunch break.
	(Aula Magna Pontecorvo)
15.00–16.00	**Camillo De Lellis.** *Regularity and structure of area minimizing currents mod p.*
16.00–16.30	Coffee break.
16.30–17.30	**Francesco Maggi.** *Soap films, soap bubbles, and almost critical points in geometric variational problems.*

	Tuesday 15 (Aula Magna Pontecorvo)
9.30–10.30	**Robert Mc Cann.** *Displacement convexity of Boltzmann's entropy characterizes positive energy in General Relativity.*
10.30–11.00	Coffee break.
11.00–12.00	**José Antonio Carrillo.** *Nonlinear aggregation-diffusion equations: (reverse) HLS inequalities and equilibration.*
	Lunch break.
15.00–16.00	**Aldo Pratelli.** *On the Sobolev approximability of planar invertible mappings.*
16.00–16.30	Coffee break.
16.30–17.30	**Ludovic Rifford.** *Recent progress in sub-Riemannian geometry.*

	Wednesday 16 (Aula Magna Pontecorvo)
9.30–10.30	**Albert Fathi.** *Singularities of solutions of the Hamilton-Jacobi equation. A toy model: distance to a closed subset.*
10.30–11.00	Coffee break.
11.00–12.00	**Nicola Gigli.** *Functional analysis and metric geometry.*
	Lunch break.
16.00–17.00	**Juan Luis Vazquez.** *Nonlinear diffusion equations driven by fractional operators.*
17.00–17.30	Coffee break.
17.30–18.30	**Antonio Corbo.** *An homogenization result with a Γ-limit absolutely continuous with respect to a singular measure.*

	Thursday 17 (Aula Magna Pontecorvo)
9.30–10.30	**Giovanni Alberti.** *On the extension of Frobenius theorem to non-smooth sets and currents.*
10.30–11.00	Coffee break.
11.00–12.00	**Guido De Philippis.** *Fine structure of measures satisfying a PDE constraint.*

Titles and abstracts

Giovanni Alberti (University of Pisa). *On the extension of Frobenius theorem to non-smooth sets and currents.*
Abstract: Given a distribution V of k-planes in $\mathbf{R}^d$, on of the implications in the classical version of Frobenius theorem can be stated as follows: if S is a regular k-dimensional surface tangent to V, then V is involutive at every point of S.
In this talk I will give an overview of some recent (and not so recent) research with A. Massaccesi (University of Verona), Evgeni Stepanov (Steklov Institute, Saint Petersburg) and Andrea Merlo (Scuola Normale Superiore, Pisa), where we investigated the extension of this statement to weaker notions of surfaces, such as rectifiable sets and currents. In particular, if S is a just a (k-rectifiable) set and not a regular surface, then the validity of the statement is strictly related to the regularity of the boundary of S. Furthermore, if S is normal current, then the key is a certain geometric property of the boundary of S. It should be noted that these questions are strictly related to the problem of decomposing normal currents into integral currents.

José Antonio Carrillo (Imperial College, London). *Nonlinear aggregation-diffusion equations: (reverse) HLS inequalities and equilibration.*
Abstract: We analyse under which conditions equilibration between two competing effects, repulsion modelled by nonlinear diffusion and attraction modelled by nonlocal interaction, occurs. I will discuss the regimes that appear in aggregation diffusion problems with homogeneous kernels. We will discuss the main qualitative properties in terms of stationary states and minimizers of the free energies. In particular, all the porous medium cases are critical while the fast diffusion are not in this regime. In the second part, I will discuss the diffusion dominated case for porous medium cases in which this balance leads to continuous compactly supported radially decreasing equilibrium configurations for all masses. All stationary states with suitable regularity are shown to be radially symmetric by means of continuous Steiner symmetrisation techniques. Calculus of variations tools allow us to show the existence of global minimizers among these equilibria. Finally, in the fast diffusion regime we are able to find conditions for the existence of global minimizers in a different range by means of reversed HLS inequalities. Concentration at the origin in part of this range is not ruled out. This talk is based on works in collaboration with S. Hittmeir, B. Volzone, Y. Yao, V. Calvez, F. Hoffmann, E. Mainini, J. Dolbeault, M. Delgadino, and R. Frank.

Antonio Corbo Esposito (University of Cassino). *An homogenization result with a Γ-limit absolutely continuous with respect to a singular measure.*
Abstract: We consider a model, one-dimensional, variational problem for a sequence of integral functionals of the type $\int_0^1 f(v')d\mu_{h,\alpha}$, where $\mu_{h,\alpha}$ is a sequence of (non coercive) measures weak* converging to the binomial measure μ_α, an orthogonal measure with

respect to the Lebesgue measure (if $\alpha \neq \frac{1}{2}$). Under suitable assumptions on f we prove the Γ-convergence of the functionals, properly scaled, to the integral functional

$$\int_0^1 f_{\hom}\left(\frac{dDv}{d\mu_{1-\alpha}}\right) d\mu_{1-\alpha}.$$

The Γ-limit is finite only on the BV functions on the unit interval absolutely continuous with respect to the binomial measure $\mu_{1-\alpha}$. The result admits some generalizations to the multidimensional case.

Guido De Philippis (SISSA, Trieste). *Fine structure of measures satisfying a PDE constraint.*
Abstract: In this talk I will present some new result concerning the structure of measure satisfying a linear PDE constraint. In 2016, in collaboration with Filip Rindler, we prove a first structural result concerning the singular part of measure subject to PDE constraint. This turned out to have several applications in GMT and in Geometric Analysis. Recently, in a joint work with Adolfo Arroyo Rabasa, Jonas Hirsch and Filip Rindler we improve upon this result proving a more precise structure on the "low" dimensional part of the measure. As a corollary we recover several known rectifiability results. In this talk I will try to give an overview of both these results and of their applications.

Camillo De Lellis (IAS, Princeton). *Regularity and structure of area minimizing currents mod p.*
Abstract: Consider the interior singular set S of an area minimizing m-dimensional current T mod p in codimension n. In the 80es White showed that, when p is odd and $n = 1$, S has dimension at most $m - 1$. Prior to his work, a similar dimension bound was only known for $p = 3$, $m = 2$ and $n = 1$ (Taylor) and for $p = 2$ (from Federer's seminal paper it follows that S has dimension at most $m - 2$). In a joint work with Hirsch, Marchese and Stuvard, we prove that the singular set S has dimension at most $m - 1$ for every p, m and n. Our proof is based on a suitable modification of Almgren's regularity theory. Combining it with the results of Naber and Valtorta, for p odd we are able to improve the dimension bound to rectifiability and finite $(m - 1)$-dimensional measure. As a corollary we achieve the following structure theorem: for p odd area-minimizing currents mod p can be decomposed into integral area-minimizing currents which meet at a common boundary.

Albert Fathi (Georgia Tech). *Singularities of solutions of the Hamilton-Jacobi equation. A toy model: distance to a closed subset.*
Abstract: This is a joint work with Piermarco Cannarsa and Wei Cheng. If $H : T^*M \to \mathbf{R}$ is a Tonelli Hamiltonian, i.e. at least C^2 convex and superlinear in the momentum, for a large class of viscosity solutions $F : M \times [0, +\infty[\to \mathbf{R}$ of the Hamilton-Jacobi equation

$$\partial_t F + H(x, \partial_x F) = 0,$$

we describe the local structure of the set $\mathrm{Sing}(F)$ of points where F is not differentiable. For example it is locally path-connected, and we will also study the homotopy type of $\mathrm{Sing}(F)$.

We will give applications in Riemannian geometry. These studies do cover the case of singularities of the Euclidean distance function $d_A : \mathbf{R}^k \to [0, +\infty[$ to a closed subset A of the Euclidean space $\mathbf{R}^k$. After considering the results in the general case, we will concentrate on the case of d_A to explain the methods of proof.

Nicola Gigli (SISSA, Trieste). *Functional analysis and metric geometry.*
Abstract: Aim of the talk is to present some aspects of the important role that functional analysis has in the context of metric geometry. I shall discuss both the case of synthetic description of lower Ricci curvature bounds, where this role is by now well understood, and some potential applications to the world of lower sectional curvature bounds, where it might potentially lead to the solution of long-standing open problems.

Francesco Maggi (University of Texas at Austin). *Soap films, soap bubbles, and almost critical points in geometric variational problems.*
Abstract: We explain how the study of almost critical points of surface energies arise in the study of the equilibrium and evolution of soap films, soap bubbles, and crystal grains. We introduce some theorems that describe (qualitatively and, sometimes, quantitatively) the possible bubbling configurations, and that provide useful criteria to exclude bubbling. This talk is based on several papers written in the past few years in collaboration with G. Ciraolo (U Palermo), M. Delgadino (Imperial College London), D. King (UT Austin), C. Mihaila (U Chicago), R. Neumayer (Nothwestern U), A. Scardicchio (ICTP Trieste), and S. Stuvard (UT Austin).

Robert McCann (University of Toronto). *Displacement convexity of Boltzmann's entropy characterizes positive energy in General Relativity.*
Abstract: Einstein's theory of gravity is based on assuming that the fluxes of a energy and momentum in a physical system are proportional to a certain variant of the Ricci curvature tensor on a smooth 3+1 dimensional spacetime. The fact that gravity is attractive rather than repulsive is encoded in the positivity properties which this tensor is assumed to satisfy. Hawking and Penrose (1971) used this positivity of energy to give conditions under which smooth spacetimes must develop singularities.
By lifting fractional powers of the Lorentz distance between points on a globally hyperbolic spacetime to probability measures on spacetime events, we show that the strong energy condition of Hawking and Penrose is equivalent to convexity of the Boltzmann-Shannon entropy along the resulting geodesics of probability measures. This new characterization of the strong energy condition on globally hyperbolic manifolds also makes sense in (non-smooth) metric measure settings, where it has the potential to provide a framework for developing a theory of gravity which admits certain singularities and can be continued

beyond them. It provides a Lorentzian analog of Lott, Villani and Sturm's metric-measure theory of lower Ricci bounds, and hints at new connections linking gravity to the second law of thermodynamics.

Aldo Pratelli (University of Pisa). *On the Sobolev approximability of planar invertible mappings.*
Abstract: This talk will be devoted to discuss the general problem of approximating planar invertible mappings with diffeomorphisms in some Sobolev sense. This is an important question in view of the applications, for instance in nonlinear elasticity, where it is useful to approximate a generic deformation with a more regular one. There are a number of crucial questions in this framework, starting from the fact that it is not obvious the correct general definition of "invertible" in this setting. The problem has attracted a lot of interest, especially in the last 15 years; several questions have obtained an answer, but some of the main ones are still open. In this talk we will present some of the most important questions, with a particular emphasis on the open ones.

Ludovic Rifford (Université Nice Sophia Antipolis). *Recent progress in sub-Riemannian geometry.*
Abstract: After a short introduction to sub-Riemannian geometry, we will present the main open problems in the theory and give an overview of recent progress.

Juan Luis Vazquez (Universidad Autónoma Madrid). *Nonlinear diffusion equations driven by fractional operators.*
Abstract: The talk presents work on the existence, regularity and typical behaviour of solutions of nonlinear fractional elliptic and parabolic equations, posed in the whole space or in bounded domains. Attention is given to functional aspects, to the boundary behaviour, and to the long time asymptotics. Work with different collaborators will be mentioned.