Non-smooth Geometry and Optimal Transport

Aula Bianchi Scienze, Scuola Normale Superiore

Pisa, 11/06/2018

Schedule

9:00/9:55. **David Tewodrose (SNS).** The Bérard-Besson-Gallot theorem on RCD(K,N) spaces

10:00/10:55. **Enrico Pasqualetto (SISSA).** On a notion of differential for metric-valued Sobolev maps.

11:00/11:55. **Gioacchino Antonelli (SNS).** The “Almost Splitting Theorem” and its importance in the study of structure results of Ricci-limit spaces.

15:00/15:55. **Ugo Bindini (SNS).** *Gamma*-convergence and Optimal Transport in Density Functional Theory.

16:00/16:55. **Federico Glaudo (SNS).** A New Approach to the Random Matching Problem in 2 Dimensions.

Abstracts

David Tewodrose

The Bérard-Besson-Gallot theorem on RCD(K,N) spaces

In 1994, P. Bérard, G. Besson and S. Gallot introduced a family of embeddings of a closed Riemannian manifold (M, g) in its space of L^2 functions. Built from the heat kernel of the manifold, this family indexed by the time variable $t > 0$ permits to define pull-back metrics g_t on M which converge to g when t goes to 0, with a first-order asymptotic expansion involving curvature terms.

After a brief exposition of the Riemannian picture, I will explain how this theorem extends to RCD(K,N) spaces, giving in particular a notion of Riemannian metric and of convergence of Riemannian metrics on such spaces.

Enrico Pasqualetto

On a notion of differential for metric-valued Sobolev maps

In this talk I will introduce a notion of differential for Sobolev maps from a metric measure space to a metric space (defined via post-composition). Such differential is given in the framework of tangent and cotangent modules over metric measure spaces, which have been developed by N. Gigli. I will prove consistency of our approach with previously known notions, first and foremost with Kirchheim's metric differential for metric-valued Lipschitz maps defined on the Euclidean space. Finally, I will discuss few possible applications of our construction. This is a joint work with Nicola Gigli and Elefterios Soultanis.

Gioacchino Antonelli

The "Almost Splitting Theorem" and its importance in the study of structure results of Ricci-limit spaces

The classical Splitting Theorem, which was proved by Cheeger and Gromoll in 1971, states that if a complete m-dimensional Riemannian manifold M satisfies $\mathrm{Ric}_M \geq 0$ and contains a line, then it splits as $\mathbb{R} \times N$ where N is a complete $(m-1)$-dimensional Riemannian manifold with $\mathrm{Ric}_N \geq 0$. In Cheeger-Colding's study of structure results of the so-called *Ricci-limit spaces*, which are metric spaces that arise as pointed Gromov-Hausdorff limit of complete m-dimensional Riemannian manifolds with a uniform bound from below on Ric, it comes out that it is useful a generalization of the classical Splitting Theorem which holds for metric spaces that satisfy $\mathrm{Ric} \geq 0$ in a *generalized* sense: namely metric spaces that arise as pointed Gromov-Hausdorff limit of complete m-dimensional Riemannian manifolds M_i that satisfy $\mathrm{Ric}_{M_i} \geq -\delta_i$, with $\delta_i > 0$ and $\delta_i \to 0$. After a brief introduction to the subject of *Ricci-limit spaces* and after having pointed out the reason for which it is useful such a generalized Splitting Theorem, we will give an outline of the proof of this theorem given by Cheeger and Colding.

Ugo Bindini

Gamma-convergence and Optimal Transport in Density Functional Theory

I will prove that the semiclassical limit of the Hohenberg-Kohn functional corresponds to the infimum of the multimarginal optimal transport problem with Coulomb cost. Moreover, I will present a new technique to rescue the marginals of a transport plan smoothed by convolution, which turns out to be the right way to attack other related open problems.

Federico Glaudo

A New Approach to the Random Matching Problem in 2 Dimensions

The random matching problem concerns the study of the transportation cost of empirical measures of independent identically distributed random variables towards their common law. Working on a 2-dimensional manifold with cost given by the quadratic Wasserstein distance W_2^2, we will illustrate an improvement over the technique by Ambrosio, Stra, Trevisan to compute the asymptotic rate of the expected cost. Furthermore, we will outline a new optimality condition for transport maps on manifolds that plays a central role in the proof. Joint work with L. Ambrosio.