ERC conference on Optimal Transport and Applications

Scuola Normale Superiore

Pisa, November 7-11, 2016

Titles and abstracts

Fabrice Baudoin (University of Connecticut).
Differential forms on Dirichlet spaces and Bakry-Emery estimates on metric graphs.

We develop a general framework on Dirichlet spaces to prove a weak form of the Bakry-Emery estimate and study its consequences. This estimate may be satisfied in situations, like the Heisenberg group or metric graphs, where generalized notions of Ricci curvature lower bounds are not available.
This is a joint work with Daniel Kelleher. `https://arxiv.org/abs/1604.02520`

Jean-David Benamou (INRIA).
Monge-Ampère Based Optimal Transportation Solvers.

Yann Brenier (Ecole Polytechnique).
Hidden convexity for some gradient flows in spaces of differential forms.

Interesting gradient flows on space of differential forms can be very simply derived from the Born-Infeld model of Electromagnetism. They inherit hidden convexity properties from the augmented BI model, leading to global existence and weak-strong uniqueness results. Some connections with higher co-dimension mean curvature flows will be mentioned.

Sergio Caracciolo (Milan University).
Random and Euclidean Bipartite Matching Problem.

I will first review the reasons of the interest in Combinatorial Optimization problems from the point of view of the Statistical Mechanics of Disordered systems. Afterwards I will concentrate on the Bipartite Matching Problem. I will present some new results on the corrections to the asymptotic behaviour with the number of points to the optimal cost, in the Random case. In the 1-dimensional Euclidean case and more generally in the d-dimension, we have informations also in the behaviour of the correlation functions. An amusing analogy with electrostatics emerges.

Pierre Cardalliaguet (Paris Dauphine University).
The master equation in mean field games.

The master equation is a kind of transport equation in the space of measures. It appears as the limit equation of systems of Hamilton-Jacobi equations associated with differential games with a large number of players. We will discuss the construction of solutions and the role of the equation in mean field game theory (mean field limit, long time behavior).

Guillaume Carlier (Paris Dauphine University).
Numerical methods for Wasserstein gradient flows.

In this talk, I will present three recent numerical methods for Wasserstein gradient flows in several dimensions. The first of these methods is purely Lagrangian and relies on displacement convexity, a discretization of the Monge-Ampre operator and tools from computational geometry (joint work with Benamou, Mrigot and Oudet). The second one is Eulerian and in the spirit of the seminal work of Benamou-Brenier (joint work with Benamou and Laborde). The last one, also Eulerian, based on an idea of Gabriel Peyré relies on entropic regularization (joint with Duval, Peyré and Schmitzer).

Maria Colombo (ETH/Zurich University).
Lipschitz Changes of Variables between Perturbations of Log-Concave Measures.

In 2000, Caffarelli showed that given a Gaussian distribution γ on R^n and a log-concave perturbation of that Gaussian distribution $e^{-U}\gamma$ with U convex, the optimal transport (for quadratic cost) that takes γ to $e^{-U}\gamma$ is a 1-Lipschitz change of variable. Motivated by his result, we exploit the relationship between optimal transportation and the Monge-Ampère equation to investigate conditions under which one can find Lipschitz changes of variables between log-concave measures, the natural generalizations of Gaussian measures, and perturbations of these measures.
This is joint work with Alessio Figalli and Yash Jhaveri.

Luigi De Pascale (Pisa University).
On multimarginal Optimal Transportation with Coulomb cost.

I will present some results on the continuity of the Coulomb Transportation cost and some estimates of it in term of several norms of the transported probability density. Then I will introduce a new approach to the calculus of the semiclassical limit of the Density Functional Theory with Coulomb interaction (which is, in fact, the Optimal Transportation Problem with Coulomb cost), this new approach allow to construct "improved" recovery sequences for 2 particles and to give a proof for 3 particles.
From joint works with G.Buttazzo and T.Champion and with U.Bindini.

Simone Di Marino (Paris-Sud University).
DFT, multimarginal optimal transport and Lieb-Oxford inequalities.

We first review the Density Functional Theory and its link with multimarginal optimal transportation. Then we will focus on the Lieb-Oxford inequality which is an estimate from below of the optimal transportation cost; we will show that this inequality is exact in the limit $N \to \infty$ for the one dimensional case.

Matthias Erbar (Bonn University).
A gradient flow approach to the Boltzmann equation.

In this talk I will show that the spatially homogeneous Boltzmann equation can be seen as the gradient flow of the entropy with respect to a suitable geometry on the space of probability measures. This geometry is given by a distance between probability measures, which takes the collision process into account. As first applications, I will discuss a novel time-discrete approximation scheme for the homogeneous Boltzmann equation and a new simple proof for the convergence of Kacs random walk, a stochastic N-particle dynamic, to the homogeneous Boltzmann equation in the limit $N \to \infty$.

Max Fathi (UC Berkeley).
Deficit estimates for the Entropy Power inequality.

The Entropy Power inequality is a general bound on the entropy of convolutions of probability measures. It was first introduced by Shannon, and later proved by Stam. Recently, Rioul gave a new proof based on transport arguments. In this talk, I will discuss how Rioul's arguments can be extended to obtain deficit estimates for the entropy power inequality in certain situations. These estimates partially extend previous results of Ball, Barthe and Naor and of Ball and Nguyen.
Joint work with Thomas Courtade (UC Berkeley).

Massimo Fornasier (Technical University of Munich).
Learning and Sparse Control of Multiagent Systems.

In the past decade there has been a large scope of studies on mathematical models of social dynamics. Self-organization, i.e., the autonomous formation of patterns, has been so far the main driving concept. Usually first or second order models are considered with given predetermined nonlocal interaction potentials, tuned to reproduce, at least qualitatively, certain global patterns (such as flocks of birds, milling school of fish or line formations in pedestrian flows, etc.). However, often in practice we do not dispose of a precise knowledge of the governing dynamics.
In the first part of this talk we present a variational and optimal transport framework leading to an algorithmic solution to the problem of learning the interaction potentials from the observation of the dynamics of a multiagent system. Moreover, it is common experience that self-organization of a society does not always spontaneously occur.
In the second part of the talk we address the question of whether it is possible to externally and parsimoniously influence the dynamics, to promote the formation of certain desired patterns. In particular, we address the issue of finding the sparsest control strategy for finite agent models in order to lead the dynamics optimally towards a given outcome. We eventually mention the rigorous limit process connecting finite dimensional sparse optimal control problems with ODE constraints to an infinite dimensional sparse mean-field optimal control problem with a constraint given by a PDE of Vlasov-type, governing the dynamics of the probability distribution of the agent population.

Wilfrid Gangbo (UCLA, Los Angeles).

Paths of minimal lengths on the set of exact differential k-forms.

We initiate the study of optimal transportation of exact differential k-forms and introduce various distances as minimal actions. Our study involves dual maximization problems with constraints on the codifferential of k-forms. When $k < n$, only some directional derivatives of a vector field are controlled. This is in contrast with prior studies of optimal transportation of volume forms ($k = n$), where the full gradient of a scalar function is controlled. Furthermore, our study involves paths of bounded variations on the set of $k$currents.

This talk is based a joint work with B. Dacorogna and O. Kneuss.

Nicola Gigli (SISSA, Trieste).

Schroedinger problem and Wasserstein geodesics.

I shall introduce the Shrödinger problem and discuss in which sense it provides a general and "good" way of approximating W_2 geodesics both in smooth and non-smooth spaces. From a work with Luca Tamanini.

Nassif Ghoussoub (British Columbia University).

Optimal Mass Transport for Ballistic Costs: Factorizations, Existence and Mean Field Games.

We investigate the optimal mass transport problem associated to a "ballistic"-related cost functional on phase space $M^* \times M$, of the form

$$b_T(v,x) := \inf\left\{\langle v, \gamma(0)\rangle + \int_0^T L(\gamma(t),\dot\gamma(t))\,dt; \gamma \in C^1([0,T),M); \gamma(T) = x\right\}, \qquad (1)$$

where $M = \mathbf{R}^d$, $T > 0$, and $L : M \times M^* \to \mathbf{R}$ is a Lagrangian that is jointly convex in both variables. Under suitable conditions on the initial and final probability measures, we establish the existence of an optimal map, by relating the problem to the fixed-end cost studied by Bernard and Buffoni, that is

$$c_T(y,x) := \inf\left\{\int_0^T L(\gamma(t),\dot\gamma(t))\,dt; \gamma \in C^1([0,T),M); \gamma(0) = x, \gamma(T) = y\right\} \qquad (2)$$

through an interpolation formula involving the Wasserstein distance. We point to a link to the theory of Mean Field Games.

Shouhei Honda (Tohoku University).

Ricci curvature and Orientability.

We discuss orientability of measured Gromov-Hausdorff limit spaces of Riemannian manifolds with Ricci curvature bounded from below, so-called Ricci limit spaces, which are typical examples of metric measure spaces having lower bounds of Ricci curvature in synthetic sense by Lott-Sturm-Villani. In order to define an orientation of a limit space, we use Cheeger-Colding theory and the regularity theory of the heat flow by Ambrosio-Gigli-Savare. Applications include relations to metric currents and a new obstruction for given closed manifold to be the cross section of a tangent cone of a noncollapsed limit space, which is closely related to Colding-Naber's characterization of tangent cones.
This talk is based on arXiv:1610.02932.

Martin Huesmann (Bonn University).

The geometry of multi-marginal Skorokhod embedding.

The martingale optimal transport problem (MOT) is a variant of the optimal transport problem where the coupling is required to be a martingale between its marginals. In dimension one, this problem is well understood for two marginals corresponding to one-step martingales.
Via the Dambis-Dubins-Schwarz Theorem the MOT can be translated into a Skorokhod embedding problem (SEP). It turns out that the recently established transport approach to SEP allows for a systematic treatment of all known solutions to (one-dimensional) MOT.
We show that the transport approach to SEP extends to a multi-marginal setup. This allows us to show that all known one-marginal solutions have natural multi-marginal counterparts. In particular (among other things), we can systematically construct solutions to genuine multi-marginal martingale optimal transport problems.
This is joint work with M.Beiglbck and A.Cox.

Kazumasa Kuwada (Tokyo Institute of Technology).

Monotonicity and Rigidity of the $\mathcal{W}$-entropy on $\mathsf{RCD}(0,N)$ spaces.

Perelman's $\mathcal{W}$-entropy plays a crucial role in his seminal work on Ricci flow. It is well-known by Perelman's entropy formula that the $\mathcal{W}$-entropy is non-increasing in time and a time derivative vanishes if and only if the space is isomorphic to a gradient shrinking Ricci soliton. L. Ni brought the notion of $\mathcal{W}$-entropy to time-homogeneous Riemannian manifolds, and the corresponding results has been studied in the literature under nonnegative Ricci curvature in an appropriate sense.
In this talk, we consider the corresponding problem on $\mathsf{RCD}(0,N)$ spaces. By following Topping's approach to this problem by optimal transport, we prove the monotonicity of the $\mathcal{W}$-entropy *without* deriving the entropy formula. Moreover, we also show a rigidity of this monotonicity. Unlike the smooth case, some other singular spaces than Euclidean spaces admit a vanishing time derivative of the $\mathcal{W}$-entropy. Our result is new even on

a weighted Riemannian manifold in the sense that we require no additional bounded geometry assumption which is used to derive the entropy formula.
This is a joint work with Xiang-Dong Li (Chinese Academy of Science).

Matthias Liero (WIAS, Berlin).
On geodesic curves and convexity of functionals with respect to the Hellinger-Kantorovich distance.

In this talk we discuss the geometric properties of the Hellinger-Kantorovich distance. The latter can be seen as the infimal convolution of the Hellinger-Kakutani distance on the set of arbitrary nonnegative and finite measures and the Kantorovich-Wasserstein distance. It arises by extending the dynamical Benamou-Brenier formulation of the Kantorovich-Wasserstein distance by an additional source term taking generation and annihilation of mass into account. We recall the basic definition of the distance and its properties. In particular, we will give a full characterization of the geodesic curves and discuss convexity conditions for functionals on the space of measures.
This is a joint work with Giuseppe Savaré and Alexander Mielke.

Jan Maas (IST, Wien).
Gradient flow structures for quantum evolution equations with detailed balance.

We present a new class of transport metrics for density matrices, which can be viewed as non-commutative analogues of the 2-Wasserstein metric. With respect to these metrics, we show that dissipative quantum systems can be formulated as gradient flows for the von Neumann entropy under a detailed balance assumption. We also present geodesic convexity results for the von Neumann entropy in several interesting situations. These results rely on an intertwining approach for the semigroup combined with suitable matrix trace inequalities.
This is joint work with Eric Carlen.

Daniel Matthes (Technical University of Munich).
A gradient flow approach to multi-component Cahn-Hilliard systems.

First, we review some of our older results on Wasserstein-like distances between vector valued measures, that are defined by means of the Benamou-Bernier formula with non-linear mobility matrices. Then we focus on those distances that correspond to tensorized Dolbeault-Nazaret-Savaré metrics, and use them to formulate nonlinear systems of degenerate fourth order parabolic equations, like the Cahn-Hilliard evolution for an arbitrary number of components, as gradient flows. We give a new proof for the existence of weak solutions using that gradient flow structure and variational techniques. Finally, we introduce a structure preserving numerical discretization and show its Gamma-convergence.
This is joint work with Jonathan Zinsl.

Andrea Mondino (ETH, Zurich).

Sectional (and intermediate Ricci) curvature lower bounds via Optimal Transport.

The goal of the seminar is to give an optimal transport characterization of lower sectional curvature bounds for smooth n-dimesional Riemannian manifolds. More generally we characterize, via optimal transport, lower (and, in some cases, upper) bounds on the so called p-Ricci curvature which corresponds to taking the trace of the Riemann curvature tensor on p-dimensional planes, $1 \leq p \leq n$. Such characterization roughly consists on a convexity condition of the p-Reny entropy along L^2-Wasserstein geodesics, where the role of reference measure is played by the p-dimensional Hausdorff measure. As application we establish a new Brunn-Minkowski type inequality involving p-dimensional submanifolds and the p-dimensional Hausdorff measure.
Joint work with Christian Ketterer (Freiburg).

Leonard Monsaingeon (Instituto Superior Tecnico, Lisboa).

Incompressible immiscible multiphase flows in porous media: a variational approach.

In this talk I will describe the competitive motion of (N + 1) incompressible immiscible phases within a porous medium as the gradient flow of a singular energy in the product of Wasserstein spaces with variable coefficients (in the spirit of S. Lisni). We show the convergence of the JKO minimizing scheme and obtain a new existence result for the physically well-established system of PDEs, consisting in the Darcy-Muskat law for each phase, N capillary pressure laws, and a total saturation constraint on the volume occupied by the fluid. Our approach does not require the introduction of unphysical global or complementary Kirchoff pressure.
This is joint work with Clément Cancès (INRIA Lille - Nord Europe RAPSODI) and Thomas Gallouët (ULG Liège)

Shin-Ichi Ohta (Kyoto University).

Some functional inequalities on non-reversible Finsler manifolds.

We consider the nonlinear analogue of the Gamma-calculus on Finsler manifolds. Although the natural Laplacian is nonlinear for Finsler manifolds, one can establish the Bochner formula (O.-Sturm, 2014) and use it to develop the Gamma-calculus. In this talk, we treat applications to functional inequalities including dimensional versions of the Poincaré(-Lichnerowicz) inequality, log-Sobolev inequality, and Sobolev inequality. This method gives us sharp estimates even for non-reversible metrics.

Mark Peletier (Eindhoven University of Technology).

Convergence of many-dislocation evolutions with multiple signs.

We prove convergence of the evolution of systems of positive and negative edge dislocations in the plane.
The many-particle limit in systems of interacting particles is a well-studied subject, and in many cases the convergence of the N-particle system to a continuum limit has been proved.

The system of this talk is special by the combination of two features: the interaction potential is singular, and the system has dislocations of both signs. The resulting evolution can be complex, and the convergence question is subtle: in other work we have showed that in some regimes convergence does not hold.
In this talk I describe a positive result, proving convergence under conditions on the scaling of the parameters. We achieve this by exploiting an elegant well-posedness framework of the limiting set of equations, established by Cannone, El Hajj, Monneau, and Ribaud in 2009. This framework makes use of a delicate combination of Orlicz-space estimates and the gradient-flow structure. Using this framework we formulate and prove a sufficient condition for the N-particle system to converge to the continuum limit.
This is joint work with Adriana Garroni, Patrick van Meurs, and Lucia Scardia.

Gabriel Peyré (Paris Dauphine University).
From Monge-Kantorovich to Gromov-Wasserstein: Numerical Optimal Transport Between Several Metric Spaces.

Optimal transport (OT) has become a fundamental mathematical theoretical tool at the interface between calculus of variations, partial differential equations and probability. It took however much more time for this notion to become mainstream in numerical applications. This situation is in large part due to the high computational cost of the underlying optimization problems. There is however a recent wave of activity on the use of OT-related methods in fields as diverse as computer vision, computer graphics, statistical inference, machine learning and image processing.
In this talk, I will review an emerging class of numerical approaches for the approximate resolution of OT-based optimization problems. These methods make use of an entropic regularization of the functionals to be minimized, in order to unleash the power of optimization algorithms based on Bregman-divergences geometry. This results in fast, simple and highly parallelizable algorithms, in sharp contrast with traditional solvers based on the geometry of linear programming. For instance, they allow for the first time to compute barycenters (according to OT distances) of probability distributions discretized on computational 2-D and 3-D grids with millions of points. This offers a new perspective for the application of OT in machine learning (to perform clustering or classification of bag-of-features data representations) and imaging sciences (to perform color transfer or shape and texture morphing). These algorithms also enable the computation of gradient flows for the OT metric, and can thus for instance be applied to simulate crowd motions with congestion constraints.
We will also discus various extensions of classical OT, such as handling unbalanced transportation between arbitrary positive measures (the so-called Hellinger-Kantorovich / Wasserstein-Fisher-Rao problem), and the computation of OT between different metric spaces (the so-called Gromov-Wasserstein problem).
This is a joint work with M. Cuturi and J. Solomon.

Filippo Santambrogio (Paris-Sud University).

Flow interchange techniques and the time-discretization of variational Mean Field Games.

First, I will briefly recall the theory of Mean Field Games, a class of games introduced by Lasry and Lions where players choose a trajectory trying to minimize a cost which could depend on the density of the other players, thus boiling down to an equilibrium issue. An important class of MFG (those which are deterministic and have a local monotone coupling) has a convex variational structure, and can be written as the minimization of an action, composed by a kinetic energy and a congestion penalization, in the class of curves in the Wasserstein space. The discretization in time makes some variational problems of the following form appear:

$$\min F(\rho) + W_2^2(\rho, \rho_{k+1}) + W_2^2(\rho, \rho_{k-1}).$$

They are very similar to a JKO scheme, up to the fact that there are two reference measures instead of one. Techniques which have been very efficient in the study of the JKO scheme, such as the so-called flow-interchange introduced by Matthes, McCann and Savaré, can be applied in this framework and allow to obtain estimates on the optimal solution ρ. In particular, one can obtain in some cases that ρ is L^∞, and I will explain why this very estimate is very important for the equivalence between minimizing the action and being an equilibrium.

Karl-Theodor Sturm (Bonn University).

Heat flow on time-dependent metric measure spaces and super Ricci flows.

We study the heat equation on time-dependent metric measure spaces (being a dynamic forward gradient flow for the energy) and its dual (being a dynamic backward gradient flow for the Boltzmann entropy). Monotonicity estimates for transportation distances and for gradients will be shown to be equivalent to the so-called dynamical convexity of the Boltzmann entropy on the Wasserstein space. For time-dependent families of Riemannian manifolds the latter is equivalent to be a super-Ricci flow.
This includes all static manifolds of nonnegative Ricci curvature as well as all solutions to the Ricci flow equation.

Allen Tannenbaum (Stony Brook University, New York).

Optimal Mass Transport and the Robustness of Complex Networks.

Today's technological world is increasingly dependent upon the reliability, robustness, quality of service and timeliness of networks including those of power distribution, financial, transportation, communication, biological, and social. For the time-critical functionality in transferring resources and information, a key requirement is the ability to adapt and reconfigure in response to structural and dynamic changes, while avoiding disruption of service and catastrophic failures. In this talk, we will outline some of the major problems for the development of the necessary theory and tools that will permit the understanding and managing of network dynamics in a multiscale manner.

Many interesting networks consist of a finite but very large number of nodes or agents that interact with each other. The main challenge when dealing with such networks is to understand and regulate the collective behavior. Our goal is to develop mathematical models and optimizational tools for treating the Big Data nature of large scale networks while providing the means to understand and regulate the collective behavior and the dynamical interactions (short and long-range) across such networks.
The key mathematical technique will be based upon the use optimal mass transport theory and resulting notions of curvature applied to weighted graphs in order to characterize network robustness. Examples will be given from biology, finance, and transportation.