Workshop "Isoperimetric Problems Between Analysis and Geometry"

Invited talks

- Franck Barthe (Toulouse)

  Isoperimetry, uniform enlargement and influences

  Abstract: This joint work with B. Huou provides a necessary and sufficient condition for an isoperimetric inequality on a probability space to hold also (up to a dimension-free constant) for the corresponding product spaces equipped with the uniform distance. This has applications to the study of geometric influences.

- Antonio Cañete (Sevilla)

  Stable hypersurfaces in Euclidean convex cones with homogeneous densities

  Abstract: Given a manifold M, we can consider a density, which is just a positive function weighting volume and perimeter functionals on M. In this talk we shall focus on convex Euclidean solid cones endowed with homogeneous densities (these densities have a nice behavior with dilations), classifying the compact stable hypersurfaces in this setting. This is part of a joint work with César Rosales (Universidad de Granada).

- Gregory Chambers (Toronto)

  The Log-Convex Density Conjecture

  Abstract: In this talk, I will explain the main components of the proof of the Log-Convex Density Conjecture. This conjecture, due to K. Brakke, asserts that balls centered at the origin are isoperimetric regions in Euclidean space endowed with a positive density which is smooth, radially symmetric, and log-convex. I will also show that these are the only isoperimetric regions, unless the density is constant on some ball.

- Eleonora Cinti (Bologna)

  A quantitative weighted isoperimetric inequality via the ABP method

  Abstract: In a recent paper X. Cabre', X. Ros and J. Serra obtain a new family of sharp isoperimetric inequalities with weights in open convex cones of $\mathbb{R}^n$. They prove that, under some concavity conditions on the weight, Euclidean balls centered at the origin (intersected with the cone) minimize the weighted isoperimetric quotient, even if the weights are nonradial —except for the constant ones. Their proof is based on the ABP method applied to an appropriate linear Neumann problem. In this talk I will present the quantitative version of these isoperimetric inequelities, whose proof is still based on the ABP method, combined with some weigthed trace inequality which allows to obtain the optimal exponent on the isoperimetric deficit. This is a joint work with X. Cabre', A. Pratelli, X. Ros and J. Serra.

- Maria Colombo (Pisa)

  Equality cases and rigidity in symmetrization inequalities

  Abstract: Isoperimetric problems are of great interest both in the Euclidean and in the Gaussian setting. A fundamental tool to understand them is the idea of symmetrizations (in the Euclidean setting, the idea is due to Steiner), as it is known that the perimeter behaves monotonically under symmetrization. Natural questions which arise in this context are then the analysis of equality cases and of rigidity for the symmetrization inequalities, where by rigidity we mean the situation when the equality cases are symmetric sets by themselves. We study a geometric characterization of rigidity. The condition is formulated in terms of a new measure-theoretic notion of connectedness for Borel sets, inspired by Federer's definition of indecomposable current. (joint work with Filippo Cagnetti, Guido De Philippis, and Francesco Maggi)

- Giovanni Franzina (Erlangen)

  The isoperimetric problem with densities

  Abstract: The question whether or not there exist isoperimetric regions with densities can be very trivial or extremely difficult to consider depending on the assumptions on the density functions. We survey some known facts about this topic and we present an existence result obtained in collaboration with Guido De Philippis (Univ. Zrich) and Aldo Pratelli (FAU Erlangen).

- Nicola Fusco (Napoli)

  Stability and minimality for a nonlocal variational problem

  Abstract: I will discuss the local minimality of certain configurations for a nonlocal isoperimetric problem used to model microphase separation in diblock copolymer melts. I will show that critical configurations with positive second variation are local minimizers of the nonlocal area functional and, in fact, satisfy a quantitative isoperimetric inequality with respect to sets that are close in L^1. As a byproduct of the quantitative estimate, one gets new results concerning periodic minimal surfaces and the global and local minimality of certain configurations.

- Michael Goldman (Leipzig)

  On isoperimetric problems with density arising from the study of Bose-Einstein condensates

  Abstract: In this talk I will present a joint work with J. Royo-Letelier concerning the derivation of a sharp interface limit for a Gross-Pitaeskii functional describing a two components Bose-Einstein condensate. We study in particular the fine properties of the effective surface tension and obtain very good agreement with predictions coming from the physical literature.

- Jimmy Lamboley (Paris Dauphine)

  Wentzell eigenvalues, Faber-Krahn inequality and stability

  Abstract: Motivated by generalized impedence boundary conditions, we consider for $\Omega \subset \mathbb{R}^d$ the first non-trivial eigenvalue for Wentzell boundary conditions:

$$\begin{cases} -\Delta u &= 0 & \text{in } \Omega \\ -\beta\Delta_\tau u + \partial_n u &= \lambda_1(\Omega) u & \text{on } \partial\Omega \end{cases}$$

where β is a nonnegative real number. This eigenvalue interpolates between the Steklov eigenvalue ($\beta = 0$) and the Laplace-Beltrami eigenvalue ($\beta = +\infty$). In the case $\beta = 0$, Brock proved a Faber-Krahn type result, namely that the ball maximizes λ_1 among domains of fixed volume. We investigate the similar question in the general case $\beta \geq 0$. To that end we generalize Brock's approach, and obtain an isoperimetric inequality involving λ_1, but which is weaker than a Faber-Krahn type result when $\beta > 0$. This is natural since a similar Faber-Krahn result for the other extremal case $\beta = +\infty$ is valid only under strong topological assumptions (Hersch inequality). Therefore, we investigate first and second order optimality conditions for this problem (maximizing λ_1 under volume constraint) and prove that the ball is a local maximum when the ambient space is of dimension 2 or 3, for any β. This is related to the question of quantitative isoperimetric inequalities, and a particular difficulty here is that the eigenvalue of the ball is not simple.

- Francesco Maggi (Austin)

  Improved convergence theorem for perimeter minimizing clusters

  Abstract: A well-known fact about sequences of perimeter almost-minimizing sets is that L^1 convergence (convergence to zero of the volume of the symmetric difference) improves to C^1 convergence (existence of boundary diffeomorphisms which converge to the identity map in C^1) whenever the limit set has smooth boundary. This is a classical application of the small excess regularity criterion, which is useful in showing the equivalence of L^1-local and C^1-local minimality conditions, as well as in proving quantitative stability inequalities, and in providing qualitative descriptions of minimizers in surface tension driven problems. The smoothness assumption on the limit set is automatically valid in dimension less or equal than 7, but may fail otherwise. Our understanding of these singularities is to lacunary to allow for an extension of the above "improved convergence theorem" when the limit set is singular. When one moves from the framework of sets to that of clusters (modeling, say, soap bubble compounds) singularities appear even in dimension 2. However, in the case of clusters, we have a very good understanding of singularities in dimension 2 and 3, based on Jean Taylor's theorem on the validity of Plateau's laws. Starting from this sharp local description of bubble clusters we explain how to prove improved convergence theorems for sequences of singular sets of perimeter almost-minimizing bubble clusters in $\mathbb{R}^2$ and $\mathbb{R}^3$, and briefly discuss some of the possible applications of these results.

- Emanuel Milman (Haifa)

  Brunn-Minkowski and Poincaré-type inequalities on weighted Riemannian manifolds with boundary

  Abstract: y systematically dualizing a generalized version of the Reilly formula (an integrated form of Bochner's formula in the presence of boundary), we obtain new Poincaré-type inequalities on a Riemannian manifold equipped with a density and on its boundary under a Curvature-Dimension condition, for various combinations of boundary conditions of the domain (convex, mean-convex) and the function (Neumann, Dirichlet). In particular, we extend and refine the Poincar'e inequalities of Lichnerowicz, Brascamp–Lieb, Bobkov–Ledoux, Nguyen and Colesanti to the weighted Riemannian setting, in a single unified framework. We then propose a new geometric evolution equation, which extends to the Riemannian setting the Minkowski addition operation of convex domains, a notion previously confined to the linear setting, and for which a novel Brunn–Minkowski inequality in the weighted-Riemannian setting is obtained. Our framework allows to encompass the entire class of Borell's convex measures, including heavy-tailed measures, and extends the latter class to weighted-manifolds having negative "dimension". Based on joint work with Alexander Kolesnikov.

- Michele Miranda (Ferrara)

Two characterizations of BV functions via semigroups in Euclidean and non–Euclidean spaces

Abstract: In this talk I shall survey some connections between the theory of semigroups and the theory of functions with bounded variation in Euclidean and non–Euclidean spaces. We shall focus our attention on two characterizations of functions of bounded variations; the first one dates from the early works of De Giorgi when the first definition and properties of functions with bounded variations and sets with finite perimeter in dimension more than one was given, the second one was suggested by Ledoux because of its connection with isoperimetric problem in Euclidean and Gaussian spaces. We shall see that these two approaches are still valid and sometimes necessary in the framework of Riemannian manifolds, metric measure spaces, Carnot groups and abstract Wiener spaces.

- Manuel Ritoré (Granada)

  Large isoperimetric regions in the product of a compact manifold with Euclidean space

  Abstract: Given a compact Riemannian manifold M without boundary, we show that large isoperimetric regions in the Riemannian product $M \times \mathbb{R}^k$ of M with the k-dimensional Euclidean space $\mathbb{R}^k$ are tubular neighborhoods of $M \times x$, $xin\mathbb{R}^k$.

- César Rosales (Granada)

  Isoperimetric and stable sets for log-concave perturbations of Gaussian measures

  Abstract: We discuss the characterization of global minimizers and second order minima of perimeter under a volume constraint in Euclidean space $\mathbb{R}^{n+1}$ for the product of an n-dimensional Gaussian density and a 1-dimensional log-concave Gaussian perturbation.

- Berardo Ruffini (Pisa)

  Isoperimetric problems with non-local penalization terms

  Abstract: The talk will focus on a model arising from the physic problem of describing the behavior of a liquid drop with a charge of $Q > 0$. Such a model takes the form

  $$\min\{P(E) + Q^2\mathcal{NL}(E) : \mathrm{vol}(E) = \text{constant}\},$$

  with $P(E)$ being the perimeter of $E \subset \mathbb{R}^N$ and $\mathcal{NL}$ a non-local operator describing the repulsive effect of the charge. We shall discuss existence issues and qualitative behaviors of minimizers depending on the value of Q and on the choice of the non-local operator. We may eventually briefly discuss a similar problem: given a charged closed wire, what shape does it take. The talk is based on collaborative work with M. Goldman and M. Novaga.

- Jean Taylor (Courant Institute, New York)

  Almgren's unfinished manuscript "Isoperimetric inequalities for anisotropic surface energie"

  Abstract : Fred Almgren's list of publications at his death in 1997 included the unfinished manuscript of the title of this talk. My objective is to state and explain the theorem and give an outline of Almgren's method of proof. I would hope that this workshop can determine whether this result has been proved by someone else during the intervening 17 years. If so, I will have given a summary of a result that people should know. If not, I hope to enlist one or more participants in helping me to complete the manuscript for publication. Briefly, the usual isoperimetric inequality states that given an oriented rectifiable closed curve C in $\mathbb{R}^3$, there exists an oriented rectifiable surface S

with boundary C such that the area of S is no greater than $1/(4\pi)$ times the square of the length of C. A much more useful version applies to *every* S with boundary C, with the integral over S of its mean curvature being added to the length of C. This sum of terms is in fact $||\delta V||(\mathbb{R}^3)$, the first variation measure of $\mathbb{R}^3$ for the varifold V associated to S. More generally, Allard's "On the first variation of a varifold" gives this version of the isoperimetric inequality for rectifiable varifolds. Almgren's manuscript seeks to prove it for any anisotropic surface energy (parametric integrand) Φ belonging to a class that he identifies,so that the mean curvature is replaced by the weighted mean curvature appropriate to Φ.

Short talks

- Elena Mäder-Baumdicker (Freiburg)

  The area preserving curve shortening flow with Neumann free boundary conditions

  Abstract: Under the area preserving curve shortening flow (APCSF), a convex simple closed plane curve converges smoothly to a circle with the same enclosed area as the initial curve (due to Gage). Note that this limit curve is the solution of the isoperimetric problem in $\mathbb{R}^2$. Corresponding to the outer isoperimetric problem for a convex domain we study the APCSF with Neumann free boundary conditions outside of a convex domain. Under certain conditions on the initial curve the flow does not develop any singularity, and it converges smoothly to an arc of a circle sitting outside of the given convex domain and enclosing the same area as the initial curve.

- Andrea Mondino (Zürich)

  Embedded surfaces of arbitrary genus minimizing the Willmore energy under isoperimetric constraint

  Abstract: The isoperimetric ratio of an embedded surface in $\mathbb{R}^3$ is defined as the ratio of the area of the surface to power three to the squared enclosed volume. The Willmore energy is defined as the L^2 norm of the mean curvature of the surface. Motivated by the Helfrich model in cell biology, we study the minimization of the Willmore energy under fixed isoperimetric ratio when the underlying abstract surface has fixed genus $g \geq 0$. This is joint work with Tristan Rivire (ETH) and Laura Keller (EPFL).

- Stefano Nardulli (Rio de Janeiro)

  TBA

  Abstract: XXX.

- Liran Rotem (Tel Aviv)

  Complemented Brunn-Minkowski inequalities

  Abstract: We discuss elementary proofs of sharp isoperimetric inequalities on a normed space equipped with a homogeneous measure. When the degree of homogeneity is positive, we use the Borell-Brascamp-Lieb inequality to present a new proof of a result by Cabre, Ros-Oton and Serra. However, it turns out that when the degree of homogeneity is negative, the relevant property is a new "Complemented Concavity" property. We will define this new notion, and explain why every homogeneous measure satisfies it. This will allow us to present a new isoperimetric inequality in the "negative" case, extending results by Canete and Rosales and by Howe. We will conclude by discussing the case of non-homogeneous measures, and explain how the homogeneity may be replaced by a suitable log-convexity assumption. Based on joint work with Emanuel Milman.