Black Swan Academy of Mathematics Conference on Geometric Analysis and PDEs

Detailed Program

Wednesday

- 09:00-10:00: Neil Trudinger (ANU)
New maximum principles and local estimates for general linear elliptic operators
We consider maximum principles and related estimates for linear second order elliptic partial differential operators in n-dimensional Euclidean space, which improve previous results, with H-J Kuo, through sharp L^p dependence on the drift coefficient b. As in our previous work, the ellipticity is determined through the principal coefficient matrix $\mathcal{A}$ lying in sub-cones of the positive cone, which are dual cones of the Garding k-cones, $k = 1, \cdots, n$. Our main results are maximum principles for bounded domains, which extend those of Aleksandrov in the case $k = n$, together with extensions to unbounded domains, depending on appropriate integral norms of $\mathcal{A}$, corresponding local maximum principles and Harnack inequalities. We also consider applications to local estimates in the uniformly elliptic case, including extensions of the Krylov-Safonov Hölder estimates and improvements of our Pucci conjecture bounds.

- 10:00-10:30: Coffee Break

- 10:30-11:30: James Stanfield (Wollongong)
TBA
TBA

- 11:30-12:30: Po Lam Yung (ANU)
Two weight estimates for difference quotients
I will explain joint work with Carlos Perez, Andreas Seeger and Brian Street, where we explore estimates for first order difference quotients of functions in $\mathbb{R}^n$ in a two-weight setting. Along the way we discuss a reverse Vitali selection algorithm, which may also be of independent interest.

- 12:30-13:30: Lunch

- 13:30-14:30: Tim Buttsworth (UNSW)
Computationally-assisted constructions of Einstein metrics
Einstein manifolds are those Riemannian manifolds for which the Ricci curvature is everywhere proportional to the Riemannian metric. Einstein manifolds are fixed points of Ricci flow, critical points of the Einstein-Hilbert action, and models for gravitational instantons; these objects are among the most important in the study of differential geometry. In this talk, I will discuss several recent results which construct and study Einstein manifolds through a combination of traditional geometric-analytic methods and techniques from the increasingly popular field of verified numerical analysis.

- 14:30-15:30: Yann Bernard (Monash)
TBA
TBA

- 15:30-16:00: Coffee Break

- 16:00-17:00: Yao Yao (NUS)
  Stability and growth for the incompressible Euler equations
  The incompressible Euler equations describe how the velocity field of an incompressible, inviscid fluid evolves over time. Even in the regimes where the global well-posedness of solutions is known, there are many open questions regarding the long-time dynamics and infinite-in-time growth of solutions. In this talk, I will discuss some recent results on the nonlinear stability of traveling wave solutions and infinite-in-time growth results.

Thursday

- 09:00-10:00: Xu-Jia Wang (Westlake)
  A new minimax principle and application to nonlinear elliptic equations
  We introduce a new minimax principle to prove the existence of multi-peak solutions to the Neumann problem of the p-Laplace equation

  $$-\varepsilon^p \Delta_p u = u^{q-1} - u^{p-1} \quad in\ \Omega,$$

  where $\varepsilon > 0$ is a small constant, Ω is a bounded domain in $\mathbb{R}^n$ with smooth boundary, $1 < p < n$ and $p < q < \frac{np}{n-p}$. The argument is based on a combination of variational method, gradient flow, and a topological degree theory. This is a joint work with Xinyue Zhang.

- 10:00-10:30: Coffee Break

- 10:30-11:30: Andrew Hassell (ANU)
  A new approach to the soliton resolution conjecture
  The soliton resolution conjecture for the nonlinear Schrödinger equation (NLS) posits that asymptotically, the 'generic' solution breaks up into a finite number of 'solitons' (solitary waves) plus 'radiation' which is the asymptotic form typical of linear solutions. I will describe a new approach to proving the soliton resolution conjecture for a class of NLS, in which the problem breaks into two parts. I will describe a solution to the second part, which is joint work with my postdocs Qiuye Jia and Xiaoxu Wu.

- 11:30-12:30: Daniel Spector (NTNU)
  β-Dimensional Spaces of Measures
  The space of finite Radon measures is a somewhat natural choice in the modeling of physical phenomena: it has a simple mathematical structure; the norm on this space can be interpreted as the mass of the physical quantity that a given measure represents; bounded sequences admit weakly-star compact through the identification of this space with the dual of the space of continuous functions which decay at infinity. Two main caveats to its use, however, are its failure to support a strong-type Sobolev inequality and its lack of stability with respect to dimensional estimates. In this talk we discuss these properties of measures in contrast to the Hardy space, and through this discussion move to a dialectical resolution: the dimension stable spaces of measures. This talk is based on joint work with Dmitriy Stolyarov and Riju Basak.

- 12:30-13:30: Lunch

- 13:30-14:30: Mark Ma (Columbia)
  Optimal Regularity for Multiple Membranes Alt-Phillips Free Boundary Problem
  In this talk we introduce an Alt-Phillips analog of the Membranes Problem. We establish Hölder regularity and the optimal regularity for its minimizers. In particular, we address

how non-linear interaction among the membranes creates difficulties compared to the endpoint cases.

- 14:30-15:30: Ramiro Lafuente (Queensland)
  Long-time behavior of collapsing Ricci flows
  In this talk, I will discuss Ricci flow solutions in arbitrary dimensions that exist for all positive times and have bounded curvature and diameter in a parabolically-natural sense. The main result is that their long-term asymptotics are governed by self-similar solutions of the flow, the so-called Ricci solitons, which satisfy an elliptic version of the flow equations. This extends to arbitrary dimensions a result of John Lott in 3D. The main difficulty is that, in general, these manifolds collapse to lower-dimensional objects, and thus doing analysis in the limit can be quite challenging. We overcome this by adapting the collapsing theory of Cheeger-Fukaya-Gromov, and in particular its further development by Naber-Tian, to the Ricci flow setting. This is joint work in progress with Francesco Pediconi (Torino).

- 15:30-16:00: Coffee Break

- 16:00-17:00: Jack Thompson (UWA)
  The halfspace theorem for nonlocal minimal surfaces
  In 1990, Hoffman and Meeks proved the strong halfspace theorem for minimal surfaces, that is any connected, proper, possibly branched minimal surface in three-dimensional Euclidean space that is contained in a halfspace must be a plane. In a joint work with Matteo Cozzi, we prove the strong halfspace theorem for nonlocal minimal surfaces. Interestingly, our result holds for hypersurfaces of any dimension, in direct contrast to the classical case which does not hold for hypersurfaces of dimension three or higher.
  In the first half of my talk, I will introduce and motivate nonlocal minimal surfaces and try to highlight some interesting similarities/differences with their classical counterparts. In the second half, I will discuss the nonlocal halfspace theorem including some ideas of the proof.

Friday

- 09:00-10:00: Jesse Gell-Redman (Melbourne)
  Wave propagators in the low mass limit
  It is a classical result of Duistermaat-Hörmander that on a Lorentzian manifold, any linear hyperbolic differential operator of order 2 (e.g. the d'Alembertian) admits four (4) local distinguished parametrices. These are locally defined approximate fundamental solutions, distinguished by the structure of their singularities, i.e. their wavefront sets.
  A central question is whether there are global exact fundamental solutions defined on a complete spacetime, i.e. whether the local parametrices above can be given by global fundamental solutions; the resolution of this question: 1) is highly dependent on global geometric features of the spacetime, 2) requires uniform global phase space analysis, that is, geometric microlocal analysis.
  In this talk we present work on this topic for the Klein-Gordon equation with mass m and the wave equation on asymptotically Minkowski spacetimes, where we obtain uniform estimates in the mass down to and including $m = 0$.
  This talk is based on various joint works with Dean Baskin (Texas A&M), Moritz Doll (U. Melbourne), Taira Kouichi (Tokyo University) and Andras Vasy (Stanford).

- 10:00-10:30: Coffee Break

- 10:30-11:30: Giorgio Poggesi (Adelaide)
  On critical points of some volume-constrained shape optimisation problems:

soap bubbles, overdetermined problems, and related questions
The talk will focus on critical points of some isoperimetric-type problems, or, more precisely, volume-constrained shape optimisation problems. The celebrated Alexandrov's Soap Bubble Theorem and Serrin's symmetry result provide characterisations of critical points for the isoperimetric problem associated with the area functional (the classical isoperimetric problem) and the isoperimetric problem associated with the torsional rigidity functional (Saint Venant's problem), respectively. We present various generalisations of these celebrated symmetry theorems and discuss related quantitative stability results.

- 11:30-12:30: João Gonçalves da Silva (Adelaide)
  A quantitative Gidas-Ni-Nirenberg-type result for the p-Laplacian via integral identities
  The study of geometric properties of solutions of Partial Differential Equations is of great interest. One of the most celebrated results in this topic is the one obtained by B. Gidas, W. M. Ni and L. Nirenberg in [Gidas, Ni, and Nirenberg, Comm. Math. Phys. *(1979)], where the authors establish radial symmetry to nonnegative solutions of the problem*

$$\begin{cases} -\Delta u = f(u) & \text{on } B, \\ u = 0 & \text{on } \partial B, \end{cases}$$

  where $B \subset \mathbb{R}^N$ (with $N \geq 2$) is a ball. Later on, several authors went on to generalize this result in several directions. In this talk, I will give an overview of the history of the problem of proving the radial symmetry of solutions to Partial Differential Equations. I will also present a quantitative version of a Gidas-Ni-Nirenberg-type symmetry result involving the p-Laplacian obtained in [Dipierro, Gonçalves da Silva, Poggesi, and Valdinoci, J. Funct. Anal. *(2025)]. Quantitative stability is achieved here via integral identities based on the proof of rigidity established by J. Serra in 2013, which extended to higher dimensions and the p-Laplacian operator an argument proposed by P. L. Lions in dimension 2 for the classical Laplacian. In passing, we obtain a quantitative estimate for the measure of the singular set and an explicit uniform gradient bound.*

- 12:30-13:30: Lunch

- 13:30-14:30: Caterina Sportelli (Granada)
  Free boundary analysis of an unwelcome invasion
  In this talk, we present a free boundary model for tumor invasion in which the evolution of the tumor boundary is coupled to cell migration, proliferation, chemotaxis, and angiogenesis. We discuss the qualitative dynamics of the evolving tumor boundary and their dependence on the model parameters.

- 14:30-15:30: Artem Pulemotov (Queensland)
  Integral Gauss formula and the Poisson equation for the G_2-Laplacian
  The study of special holonomy involves a nonlinear second-order operator on differential forms called the G_2-Laplacian. In the first half of the talk, we will discuss a formula linking this operator to a natural Hodge Laplacian on a hypersurface. This result bears intriguing resemblance to the Gauss–Codazzi equation for the scalar curvature. In the second half, we will explain how our formula provides an integrability condition for the Poisson equation associated with the G_2-Laplacian in the presence of cohomogeneity one symmetry. Joint work with Timothy Buttsworth (The University of New South Wales) and Stepan Hudecek (The University of Queensland).

- 15:30-16:00: Coffee Break

- 16:00-17:00: Mat Langford (ANU)
  Conformal contraction of surfaces by powers of curvature
  We study the deformation of surfaces by powers $\alpha > 0$ of their curvature, generalizing the Ricci–Yamabe flow (corresponding to $\alpha = 1$). However, when $\alpha \neq 1$, the flow is parabolic only when the curvature is positive, and gives rise to a non-linear reaction-diffusion equation for the curvature which resembles the fast diffusion equation when $\alpha < 1$, and the porous medium equation when $\alpha > 1$. As such, the flow exhibits "fast-diffusion" characteristics (instantaneous smoothing, infinite propagation of positive curvature) when $\alpha < 1$ and "slow-diffusion" characteristics (delayed regularity, finite propagation of positive curvature) when $\alpha > 1$. We classify the soliton solutions for the equation and establish a differential Harnack inequality for the "pressure function", a monotonicity formulae for the entropy, and a monotonicity formula for the conformal energy, which we are able to exploit to establish the convergence of solutions to round points for all powers $\alpha > 0$. This is joint work with Steven Buchanan.