Celebrating the 25th anniversary of Calculus of Variations and Partial Differential Equations

Centro De Giorgi, Scuola Normale Superiore Pisa, May 18, 2018

Schedule (preliminary)

9.00-9.10. Welcome address
9.10-9.30. Report on the journal by **J.Holland (Springer).**
9.20-10.10. **J.Jost (MPI Leipzig).** *Quantum field theory and geometric analysis.*
Coffee break.
10.40-11.30. **G.Buttazzo (Pisa).** *One dimensional optimal reinforcements of elastic structures.*
11.40-12.30. **M.Struwe (ETH, Zurich).** *Harmonic 3-spheres, or normalized harmonic map heat flow.*
Lunch break
14.30-15.20. **G.De Philippis (SISSA).** *Boundary Regularity for Mass Minimising currents.*
Coffee break.
15.40-16.30. **G.Alberti (Pisa).** *On the structure of minimal 2-dimensional N-partitions (and N-clusters) for large N.*

Abstracts

Alberti. By minimal N-partitions I mean partitions of a given bounded 2-dimensional domain E which consist of N sets (cells) with equal area, and which minimize the length of the union of the boundaries of these sets. Similarly, minimal N-cluster are families of N pairwise disjoint sets with equal prescribed areas, which again minimize the length of the union of the boundaries. Among other results, T. C. Hales proved in 2001 that if E is a flat 2-dimensional torus then the regular hexagonal partition (when it exists) is the only minimal N-partition. Apart from this result, not much is known about the structure of N-partitions when N is large. In particular when E is a planar domain we expect that minimizing N-partitions should look hexagonal at least in some asymptotic sense. Similar issues arise in the study of the asymptotic shape of minimal N-clusters as N tends to infinity. In this talk I will describe a few results obtained so far and future directions. This is a work, still in progress, with Marco Caroccia (University of Lisbon) and Giacomo Del Nin (University of Pisa).

Buttazzo. We study the optimal reinforcement of an elastic membrane, fixed at its boundary, by means of a connected one-dimensional structure. The problem consists in finding the optimal configuration for the stiffeners, the problem is then a shape optimization problem, where the admissible competing shapes are one-dimensional networks of prescribed length. We show the existence of an optimal solution that may present multiplicities, that is regions where the optimal structure overlaps. The case where the connectedness assumption is removed is also presented. Some numerical simulations are shown to confirm the overlapping phenomenon and to illustrate the complexity of the optimal structures when their total length becomes large.

De Philippis. Federer and Fleming integral currents allow to solve the Plateau problem in arbitrary Riemannian manifolds in any dimension and co-dimension. Thanks to the monumental work of Almgren (recently revised by De Lellis and Spadaro) interior regularity is by now quite well understood. On the other hand, the current literature fails to provide (for the high co-dimension case) even a single regular point at the boundary unless we require rather restrictive assumptions on the ambient space. In this talk I will give an overview of the problem and show a first boundary regularity result for mass minimising currents in any co-dimension. In particular, I will show that regular points are dense in the boundary. This, among other things, allows to provide a positive answer to a question of Almgren, namely that for connected boundary data, the solution is actually connected and of multiplicity one.
This is a joint work with C.De Lellis, J.Hirsch and A.Massaccesi.

Jost. I shall describe how models from quantum field theory, like the (supersymmetric) nonlinear sigma model, can be converted into geometric variational problems. Since these problems typically possess a noncompact invariance group, standard variational schemes do not apply. On the other hand, however, via Noether's theorem, these symmetries provide us with additional structure that help us with the analytical treatment. I shall argue that these problems, because of their rich and difficult structure, can play a pioneering for the development of new techniques in geometric analysis.

Struwe. Finding non-constant harmonic 3-spheres for a closed target manifold N is a prototype of a super-critical variational problem. In fact, the direct method fails, as the infimum of Dirichlet energy in any homotopy class of maps from the 3-sphere to any closed N is zero; moreover, the harmonic map heat flow may blow up in finite time, and even the identity map from the 3-sphere to itself is not stable under this flow.
To overcome these difficulties, we propose the normalized harmonic map heat flow as a new tool, and we show that for this flow the identity map from the 3-sphere to itself now, indeed, is stable; moreover, the flow converges to a harmonic 3-sphere also when we perturb the target geometry. While our results are strongest in the perturbative setting, we also outline a possible global theory, which may open up a rich research agenda.