Minimal energy solutions and bifurcation results for a weakly coupled nonlinear Schrödinger system

In this talk I intend to give a panoramic view on existence results for fully nontrivial solutions of the nonlinear Schrödinger system

$$\begin{aligned} -\Delta u + \quad u &= u^3 + buv^2 \qquad \text{in } \mathbb{R}^n, \\ -\Delta v + \omega^2 v &= v^3 + bvu^2 \qquad \text{in } \mathbb{R}^n, \\ u, v \in H^1(\mathbb{R}^n), \quad n &\in \{1,2,3\}, \quad \omega > 0, b \in \mathbb{R} \end{aligned}$$

which have been obtained during the past ten years. The focus will be set on a comparison of the methods coming from bifurcation theory and constrained minimization techniques respectively critical point theory. Amongst other things I will show that in case $n \in \{2,3\}$ positive solutions exist and converge to a solution of some optimal partition problem as the coupling parameter b tends to $-\infty$ whereas this phenomenon does not occur when $n = 1$.