

Existence and non existence results for the singular Nirenberg problem

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Abstract. In this talk we will study the problem of prescribing Gaussian curvature on compact surfaces under conformal change of the metric with conical singularities. We restrict our attention to the case of the unit sphere, which in the regular version is known as the Nirenberg problem. Let be a smooth function $K : \mathbb{S}^2 \rightarrow \mathbb{R}$, a set of points $\{p_1, \dots, p_m\}$ in which are located conical singularities with orders $\{\alpha_1, \dots, \alpha_m\}$ and $m \in \mathbb{N}$, the equation

$$-\Delta_0 u = \lambda \left(\frac{K e^u}{\int_{\mathbb{S}^2} K e^u dV_{g_0}} - \frac{1}{4\pi} \right) - 4\pi \sum_{j=1}^m \alpha_j \left(\delta_{p_j} - \frac{1}{4\pi} \right) \quad \text{in } \mathbb{S}^2, \quad (1)$$

where λ is a positive parameter, solves the problem. The case with positive K has been wide studied, (see for instance [2]), so we focus on the case of sign-changing K , which presents some difficulties in the compactness argument. In order to find solutions of (1), we employ variational and min-max methods. Besides, we present different cases where there is non existence for (1).

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References

- [1] F. De Marchis, R. López-Soriano. Existence and non existence results for the singular Nirenberg problem. in preparation.
- [2] A. Malchiodi, D. Ruiz, New improved Moser-Trudinger inequalities and singular Liouville equations on compact surfaces, *Geom. Funct. Anal.* 21 (2011), no. 5, 1196–1217.

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