SCUOLA
NORMALE
SUPERIORE

SEMINARIO DI MATEMATICA

Martedì 31 marzo 2015
ore 16:00

Scuola Normale Superiore
Pisa
Aula Mancini

Fabrizio Catanese

University of Bayreuth

Terrà un seminario dal titolo:

"Higher dimensional lemniscates: m particles in n-space with logarithmic potentials via complex analysis"

Abstract:

What is a lemniscate? Given m points w_j in n-dimensional space, consider the function F(x) which is the product of the distances |x-w_j|. A lemniscate is defined to be a singular level set of the function F, and the configuration of lemniscates is said to be generic if f= log(F) is a global Morse function. In my work with Paluszny 25 years ago we completely classified the possible lemniscate configurations in the plane, showing that there is a bijection with the set of connected components (`chambers') of the open set of the configuration space which parametrizes the Morse functions. And we showed that the number of configurations leads to the generating function (1-sin t)^{-1}, the same which was found by Arnold for Morse functions in one real variable. With Ingrid Bauer we extended the results to Riemann surfaces of higher genus, using the tight connection with Riemann's existence theorem, and braid group actions. Recently, together with Bauer and Di Scala, we made a breakthrough in higher dimensions, showing via elementary complex analysis that the critical points of F have Hessian of positivity at least (n-1). This implies that if F is a Morse function, then it has only local minima and saddle points with negativity 1. The critical points lie in the convex span of the points w_j (these are absolute minima): but we made the discover that F can also have other local minima (we show this by using symmetry and symmetry breaking). The chambers in configuration space are divided by two type of walls, those of quantitative type (where the number of local minima changes), and those of qualitative type (where the configuration may change). Major open questions are: bounding effectively the number of minima in terms of the dimension n and the number of points m, finding generating functions for the number of configurations, and the number of chambers in configuration space. The study of analogous questions with different potentials is also an interesting question.

Piazza dei Cavalieri, 7
56126 Pisa-Italy
tel: +39 050 509111
fax: +39 050 563513

23/03/2015