

## Uniform bounds for strongly competing systems: the optimal Lipschitz case

We present uniform regularity results regarding family of positive solutions of

$$\begin{cases} -\Delta u_{i,\beta} = f_{i,\beta}(u_{i,\beta}) - \beta u_{i,\beta} \sum_{j \neq i} a_{ij} u_{j,\beta}^p & \text{in } \Omega \\ u_{i,\beta} = 0 & \text{on } \partial\Omega \end{cases} \quad i = 1, \dots, k, \beta > 0$$

in the cases  $p = 1$  (symmetric interaction) and  $p = 2$  (variational interaction). For such systems, of interest in population dynamics and in the study of phase-separation of Bose-Einstein condensates, we show that  $L^\infty(\Omega)$ -boundedness implies  $\mathcal{C}_{loc}^{0,1}(\Omega)$ -boundedness, uniformly in  $\beta \rightarrow +\infty$ . This extends the  $\mathcal{C}^{0,\alpha}$ -regularity theory available in the literature ( $0 \leq \alpha < 1$ ) to the optimal Lipschitz case.

This is a joint work with Alessandro Zilio (Centre d'Analyse et de Mathématiques Sociales, Paris).