

Uniform bounds for strongly competing systems: the optimal Lipschitz case

We present uniform regularity results regarding family of positive solutions of

$$\begin{cases} -\Delta u_{i,\beta} = f_{i,\beta}(u_{i,\beta}) - \beta u_{i,\beta} \sum_{j \neq i} a_{ij} u_{j,\beta}^p & \text{in } \Omega \\ u_{i,\beta} = 0 & \text{on } \partial\Omega \end{cases} \quad i = 1, \dots, k, \quad \beta > 0$$

in the cases $p = 1$ (symmetric interaction) and $p = 2$ (variational interaction). For such systems, of interest in population dynamics and in the study of phase-separation of Bose-Einstein condensates, we show that $L^\infty(\Omega)$ -boundedness implies $\mathcal{C}_{loc}^{0,1}(\Omega)$ -boundedness, uniformly in $\beta \rightarrow +\infty$. This extends the $\mathcal{C}^{0,\alpha}$ -regularity theory available in the literature ($0 \leq \alpha < 1$) to the optimal Lipschitz case.

This is a joint work with Alessandro Zilio (Centre d'Analyse et de Mathématiques Sociales, Paris).