Abstract

We discuss results concerning an elliptic equation of Liouville type, modeling the behavior of selfgravitating cosmic strings. More specifically, we consider the following problem:

$$\begin{cases} -\Delta u = e^{au} + |x|^{2N} e^u \quad \text{in } \mathbb{R}^2 \\ \int_{\mathbb{R}^2} \left(e^{au} + |x|^{2N} e^u \right) < \infty \end{cases} \tag{1}$$

with $a > 0$ and $N > -1$.

Problem (1) furnishes a degenerate case of more general system of Liouville type, that we shall discuss as time permits.

For a solution u of (1) the value:

$$\beta := \frac{1}{2\pi} \int_{\mathbb{R}^2} \left(e^{au} + |x|^{2N} e^u \right) \tag{2}$$

relates to the (finite) energy of the corresponding string configuration, and our main concern will be to identify the (sharp) range of β's for which (1)+(2) is solvable. Typically it forms a bounded interval, whose extremals we are able to identify via a blow–up analysis, in terms of the parameters a and N.

The blow–up analysis will also help us to deduce symmetry results (e.g. radial symmetry), in some limiting cases.