Dipartimento di Matematica
Seminario di Equazioni Differenziali

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Regularity of the extremal solution of $-\Delta u = \lambda f(u)$

We will consider the reaction diffusion equation $-\Delta u = \lambda f(u)$ with zero Dirichlet boundary condition, where $\Omega \subset \mathbb{R}^n$ is a smooth bounded domain, λ is a positive parameter, and the reaction term $f \in C^1(\mathbb{R})$ is an increasing function satisfying $f(0) > 0$ and $f(t)/t \to +\infty$ as $t \to +\infty$. Under these assumptions it is well known that there exists an extremal parameter $\lambda^\star \in (0, +\infty)$ such that this problem admits a classical (minimal) solution u_λ for $\lambda \in (0, \lambda^\star)$ and admits no weak solution for $\lambda > \lambda^\star$. The increasing limit $u^\star := \lim_{\lambda \uparrow \lambda^\star} u_\lambda$ is known to be a weak solution for $\lambda = \lambda^\star$ and it is called the extremal solution. The regularity of $u^\star$ has been an active topic of research in the last decade. However for general reaction terms f and general domains Ω the known results are far to be optimal.

In this talk I will introduce some of the results available in the literature concerning the regularity of the extremal solution and I will concentrate my attention in some L^q and $W^{1,q}$ estimates obtained recently in [1] and [2].

References

[1] X. Cabré and M. Sanchón, Geometric-type Sobolev inequalities and applications to the regularity of minimizers, J. Funct. Anal. **264** (2013), 303-325.

[2] M. Sanchón, $W^{1,q}$ estimates for the extremal solution of reaction-diffusion problems, Preprint arXiv:1209.1526 (2012). To appear in Nonlinear Anal.