One day workshop on some recent trends in Analysis

1 February 2012, Aula Magna, Dipartimento di Matematica "L. Tonelli", Pisa

Programme:

10.30 Luigi Ambrosio (*Scuola Normale Superiore, Pisa*)

"Existence of Eulerian solutions to the semigeostrophic equations in physical space: the 2-dimensional periodic case"

11.45 Juan Casado-Díaz (Universidad de Sevilla)

"Homogenization of the membrane and Stokes problems in dimension 2 with coefficients in L^1"

16.00 Stefano Bianchini (*S.I.S.S.A.*, Trieste)

"On Sudakov's decomposition of Monge's transportation problem"

17.30 François Murat (Université Pierre et Marie Curie, Paris)

"Finite elements approximation of second order linear elliptic equations in divergence form with right-hand side in L^1"

Organizers: G. Buttazzo, F. Colombini, M.S. Gelli

Abstracts of the talks.

Ambrosio: We use new regularity and stability estimates for Alexandrov solutions to Monge-Ampere equations, recently established by De Philippis and Figalli, to provide global in time existence of distributional solutions to the semigeostrophic equations on the 2-dimensional torus, under very mild assumptions on the initial data. A link with Lagrangian solutions is also discussed.

Bianchini: We will present a proposal of what the decomposition of transportation problems with norm cost proposed by Sudakov could be. Our proposal enjoys the following properties:1) every component is essentially ciclically connected (in the sense of optimal transportation); 2) the decomposition is in many cases independent of the transport problem; 3) it can be applied to singular 1-homogeneous costs.

Casado Diaz: We deal with the homogenization of rigid heterogeneous plates. Assuming that the coefficients are equi-bounded in L^1, we prove that the limit of a sequence of plate equations remains a plate equation which involves a strongly local linear operator acting on the second gradients. This compactness result is based on a div-curl lemma for fourth-order equations. On the other hand, using an intermediate stream function we deduce from the plates case a similar result for high-viscosity Stokes equations in dimension two, Finally, we show that the L^1-boundedness assumption cannot be relaxed.

Murat: We consider, in dimension $d\ge 2$, the standard P1 finite elements approximation of the second order linear elliptic equation in divergence form with coefficients in $L^\infty(\Omega)$ which generalizes Laplace's equation. We assume that the family of triangulations is regular and satisfies the discrete maximum principle. When the r-h. s. belongs to $L^1(\Omega)$, we prove that the unique solution of the discrete problem converges in $W^{1,q}_0(\Omega)$, $1\le q<d/(d-1)$, to the unique renormalized solution of the problem. We obtain a weaker result when the r-h.s. is a bounded Radon measure. In the case where the dimension is $d=2$ or $d=3$ and where the coefficients are smooth, we give an error estimate in $W^{1,q}_0(\Omega)$ when the right-hand side belongs to $L^r(\Omega)$ for some $r>1$.