Wodzicki Residue and Dirac Operators on $\mathbb{R}^n$

UBERTINO BATTISTI
Università degli Studi di Torino

In 1984 M. Wodzicki introduced a trace on the algebra of classical pseudodifferential operators acting on a closed (compact with empty boundary) manifold. This result was first obtained from studies about the function $\zeta(A, z)$ of elliptic operators, later was given another definition using the theory of simpletic cones. This new tool was used in different fields of mathematics. A remarkable application was proved by Kastler-Kalau-Waltze in 1995. They proved that on a closed manifold, of dimention 4, with a spin structure holds the equality

$$res(D^{-2}) = C \int_M \mathscr{L}_g dv, \tag{0.0.1}$$

where D^{-2} is a parametrix of the Dirac operator and $\mathscr{L}_g$ is the Lagrangian of the Einstein-Hilbert action, that is the scalar curvature. In the following years appeared others proves of the this result using the link between Heat Kernel expansion of D^2 and $\zeta(D^2, z)$. We will extend this result to $\mathbb{R}^4$ endowed with a suitable metric. The main problems we have to solve are:

i) Wodzicki residue is just defined for closed manifold.

ii) On $\mathbb{R}^n$ Einstein-Hilbert action can be non convergent.

iii) In the equation (0.0.1) D^{-2} is a parametrix modulo smoothing operators w.r.t. ξ.

Using the operator res_ψ, defined by F. Nicola in 2003, we will prove that, endowing $\mathbb{R}^4$ with an asymptoticaly flat metric, that is a metric such that

$$g(x) = I + O(|x|^{-\alpha}) \quad \alpha > 2, |x| > R.$$

the result (0.0.1) still holds

$$\mathrm{res}_\psi(D^{-2}) = C \int_{\mathbb{R}^4} \mathscr{L}_g dv.$$