

INTENSIVE RESEARCH TRIMESTER
"GEOMETRIC FLOWS AND GEOMETRIC OPERATORS"
PISA, SPRING 2009

ZINDINE DJADLI, ANDREA MALCHIODI, AND CARLO MANTEGAZZA

The trimester will be hosted and organized by the Centro di Ricerca Matematica "Ennio De Giorgi" in Pisa, Scuola Normale Superiore, during May, June and July 2009, in collaboration with INdAM and Institut Joseph Fourier, Grenoble (FR).

Website: <http://cogmt.sns.it/GFO>

1. PRESENTATION OF THE TRIMESTER

The trimester will concentrate on two main topics, related to series of lectures at introductory level.

The first one is the study of geometrically invariant operators, possibly of higher order. In recent years, there has been an intensive study of the relations between this kind of operators (which satisfy some invariance property under conformal change of metric), their associated scalar invariants and the study of the related partial differential equations. In two dimensions for example, the integral of the Gauss curvature (whose transformation law under conformal changes of metrics is related to the Laplace operator) determines completely the topology of the surface. In four dimensions there are partial analogues, involving the Q -curvature and the Paneitz operator, but most results are limited to the case of manifolds with positive curvature: it would be interesting to relax these assumptions and consider more general situations.

Lectures related this research direction will be given by Matt Gursky, Jeff Viaclovsky and Andrea Malchiodi.

The second topic will concern geometric flows, in particular, higher order flows, "coupling" of flows and relations to theoretical physics.

Klaus Ecker will introduce the mean curvature flow in his series of lectures, with special attention to regularity theory and the connections with Ricci flow.

Lei Ni will present recent progresses in applying Ricci flow to study the structure of Riemannian and Kähler manifolds.

Mauro Carfora will discuss some recent ideas in setting connections between geometric flows and theoretical physics, in particular with quantum field theories, string theory and renormalization group techniques.

A workshop putting together leading theoretical physicists and mathematicians is planned for the second part of June.

The recent results about Ricci flow gave a strong impulse to the field of evolution of geometric structures, but not so much is known at the moment about other "natural" flows which

can be used analogously to find out canonical metrics or to investigate the geometry and topology of manifolds, in particular in higher dimensions.

One of the goal of the trimester is to try to generalize these techniques, considering other flows associated to geometric quantities like functionals of the curvatures, with special care for the ones which are conformally invariant, for instance, the Q -curvature.

This analysis leads to higher order flows and it is one of our motivations to put together people working on higher order differential operators and on geometric flows.

At the end of June there will be the general conference of the trimester.

We plan, in parallel to the series of lectures, to organize daily seminars given by the guests at the Center De Giorgi. All the people will be invited to give a talk on their research activities.

Financial support is available for young students. Interested people need to apply at the website of the trimester.

The official website of the trimester is <http://cvgmt.sns.it/GFO>, where you can find information about registration and application forms.

There is a mailing list for participants or interested people to get up-to-date information on the trimester at <http://cvgmt.sns.it/gfopisa>

If you have problems in registering to the list, please write an email to c.mantegazza@sns.it, possibly putting GFO in the subject.

There will be some possibly related events for interested people, during the semester

- *Indam School on symmetry for elliptic PDEs, Roma Univ "La Sapienza", May 25–29, 2009*
<http://www.mat.uniroma2.it/valdinoc/scuola-indam.html>
Lectures by Xavier Cabré, Manuel del Pino, Ermanno Lanconelli and Ovidiu Savin
- *Fifth Summer School in Analysis and Applied Mathematics*
Rome – Univ. "La Sapienza", June 1–5, 2009
<http://www.mat.uniroma1.it/people/garroni/School2009.html>
Lectures by Giovanni Alberti, Stefan Hildebrandt, László Székelyhidi Jr.

2. SCIENTIFIC COMMITTEE

Luigi Ambrosio – SNS, Pisa

Gérard Besson – Institut Joseph Fourier, Grenoble

Jean-Pierre Bourguignon – IHES, Bures-sur-Yvette

Sun-Yung Alice Chang – Princeton

Gerhard Huisken – Max-Planck-Institut für Gravitationsphysik, Albert Einstein Institute, Golm

3. ORGANIZING COMMITTEE

Luigi Ambrosio – SNS, Pisa

Zindine Djadli – Institut Joseph Fourier, Grenoble

Carlo Mantegazza – SNS, Pisa

Andrea Malchiodi – SISSA, Trieste

4. MAIN CALENDAR

<i>Week</i>	<i>Events</i>
May	
Monday 4 – Friday 8	Gursky Lectures I
Monday 11 – Friday 15	Gursky Lectures II
Monday 18 – Friday 22	Carfora Lectures I
Monday 25 – Friday 29	Carfora Lectures II
June	
Monday 1 – Friday 5	—
Monday 8 – Friday 12	Malchiodi/Viaclovsky Lectures
Monday 15 – Friday 19	Ni Lectures
Monday 22 – Friday 26	Workshop on GeoFlows in Math/Physics
Monday 29 – Friday 3	General Conference “GFO in Pisa”
July	
Monday 6 – Friday 10	Ecker Lectures I
Monday 13 – Friday 17	Ecker Lectures II
Monday 20 – Friday 24	—

5. ABSTRACTS OF THE LECTURES

Matt Gursky, University of Notre Dame – *Fully Nonlinear Equations in Conformal Geometry*

Our goal in this series of lectures is to provide a comprehensive introduction to the σ_k -Yamabe problem, and some aspects of the general theory and applications of fully nonlinear equations in conformal geometry.

Mauro Carfora, University of Pavia – *Ricci Flow in Theoretical and Mathematical Physics*

Ricci flow plays a natural role in many distinct physical theories. In this series of lectures we review a few case studies which characterize an important interaction between mathematics and physics. In particular, we shall discuss the relations between Ricci flow and non-linear sigma model theory, the applications of Ricci flow to general relativity, and finally some aspects of the role of Ricci flow in the theory of diffusion processes on Riemannian manifolds.

Andrea Malchiodi, SISSA, Trieste – *Functional Determinants and Conformal Geometry*

Conformal variations of renormalized determinants for covariant operators can be sometimes expressed by explicit formulas. It is natural then to find metrics which might extremize the determinants: for example in two dimensions this is a way to characterize constant

curvature metrics. I will discuss the derivation of such formulas and some cases in which it is possible to find or characterize extremal metrics.

Jeff Viaclovsky, University of Wisconsin, Madison – *Geometry of Anti-Self-Dual Metrics*

I will begin with some basic notions in 4-dimensional Riemannian geometry in order to define the concept of half-conformally-flat metrics (which are also known as self-dual or anti-self dual metrics), and a generalization of these known as Bach-flat metrics. These equations are elliptic in a suitable gauge, and I will discuss a basic regularity theorem. I will also discuss volume growth and Cheeger-Gromov convergence of such metrics, and discuss some explicit examples.

Lei Ni, University of California, San Diego – *Geometric Applications of Ricci Flow*

Ricci flow has been proven powerful in study the topology and geometry of manifolds by the fundamental work of Hamilton and Perelman. We shall focus on some recent progresses in applying Ricci flow to study the structure of Riemannian and Kähler manifolds.

Klaus Ecker, Freie Universität, Berlin – *Regularity Theory for Mean Curvature Flow*

I will review methods used in the study of singularities for mean curvature flow following mainly my book on regularity theory but I will also discuss new developments related to Perelman's work for Ricci flow.

- K. Ecker, *Regularity theory for mean curvature flow*. Progress in Nonlinear Differential Equations and their Applications, 57. Birkhäuser Boston, Inc., Boston, MA, 2004.
- K. Ecker, *A formula relating entropy monotonicity to Harnack inequalities*. Comm. Anal. Geom. 15 (2007), no. 5, 1025–1061.

6. WORKSHOPS

Geometric Flows in Mathematics and Theoretical Physics – 22/25 June 2009

The workshop aims to report on the recent interdisciplinary developments about geometric flows with an emphasis on the discussion of problems and new research directions coming from the mathematical and theoretical physics communities.

Ricci flow has been the point of departure and the motivating example for important developments in geometric analysis, most spectacularly for G. Perelman's proof of Thurston's geometrization program for three-manifolds and of the famous Poincaré conjecture. As it is well known, the Ricci flow also appears naturally in Quantum Field Theory (QFT), as the weak coupling limit of the renormalization group analysis of nonlinear sigma models. The existence of a QFT avatar is by no means an exclusive of Ricci flow, but it is a property shared by the mean curvature flow, by the curve shortening flow, and many other (weakly) parabolic flows of geometrical nature. Such an observation suggested that methods of quantum field theory may have useful applications in geometric flow theory, and has been the motivating stimulus of a remarkable series of meetings among mathematicians and physicists working in geometric analysis and in quantum field theory. The first of such meetings was

held at the Albert Einstein Institute in Potsdam in November 2006. That meeting was very successful and set an agenda that in a year span has already seen other two very stimulating workshops in Banff and Munich. Many ideas have been put forward during these meetings and it is becoming increasingly apparent that the interaction between mathematicians and physicists in this area is indeed very fruitful. The fourth in this series of workshops will be held in Pisa at the Centro De Giorgi of the Scuola Normale Superiore from the 22nd to the 25th of June 2009. As in the previous meetings, our objective is to provide a fast and informal channel of communications between the two communities, suggesting new problems of common interest and possible strategies of solutions. The properties of geometric flows discussed in depth by the methods of geometric analysis may shed light on the nature of significant models of QFT, whereas the imagination of QFT, often transcending and transgressing formal boundaries, may suggest new powerful strategies in geometric flow theory.

Speakers will include:

*J. Bakas, G. Bellettini, H.-D. Cao**, *F. Costantino, D. Friedan, J. Gegenberg, D. Glickenstein, J. Hoppe, J. Isenberg, A. Magni, S. Maillot, L. Ni**, *M. Petropoulos, M. Troyanov, E. Tsatis, V. Suneeta, E. Woolgar*

*To be confirmed

For more information, see at <http://cogmt.sns.it/MPRicci-Workshop>
or contact Mauro Carfora at Mauro.Carfora@pv.infn.it

General Conference “GFO in Pisa” – 29 June/3 July 2009

Speakers will include:

*L. Bessieres**, *R. Müller, M. Novaga, H. Seshadri, W. Sheng, C. Sinestrari**, *M. Struwe, G. Wang* ...*

*To be confirmed

For more information, see at <http://cogmt.sns.it/GFO-Conf>
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