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Infinite dimensional harmonic analysis and probability

One of the main problem in harmonic analysis is to describe the set $\mathfrak{P}$ of invariants kernels of positive type on a homogeneous space X, for the action of a group G. The set $\mathfrak{P}$ is a convex cone, and the first step is to determine the extremal rays, and the next step is to prove that any kernel in $\mathfrak{P}$ has an integral representation over $\mathrm{Ext}(\mathfrak{P})$.

If $X = \mathbb{R}^n$, and $G = \mathbb{R}^n$ acting by translations, then an invariant kernel K has the form

$$K(x, y) = \varphi(x - y),$$

where φ is a function of positive type. The problem is then solved by the theorem of Bochner. If $X = \mathbb{R}^n$, and $G = O(n) \ltimes \mathbb{R}^n$ is the motion group, then the integral representation involves Bessel functions.

For an infinite dimensional Hilbert space such a problem has been considered many years ago by Schoenberg (1938) and Krein (1949). More recently this problem knew striking developpments with the works of Voiculescu, Vershik, Olshanski, Pickrell, Borodin, in relationship with the very activ research domain of random matrices. One of the main results by Olshanski and Vershik is about the space of infinite Hermitian matrices with the action of the infinite unitary group. Olshanski introduced the notion which extends the classical one of Gelfand pair. In fact a spherical pair (G, K) is an inductive limit of a sequence of Gelfand pairs $\big(G(n), K(n)\big)$. On one side one discovers in this new harmonic analysis phenomenons which don't appear in the finite dimensional case, as the multiplicativity property of the spherical function. On the other side it involves classical analysis as orthogonal polynomials, totally positive functions, or entire functions of the Laguerre class which were considered by Pólya.

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