Sobolev-Lorentz spaces in the Euclidean setting and counterexamples

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Abstract

This paper studies the inclusions between different Sobolev-Lorentz spaces $W^{1,(p,q)}(\Omega)$ defined on open sets $\Omega \subset \mathbf{R}^n$, where $n \geq 1$ is an integer, $1 < p < \infty$ and $1 \leq q \leq \infty$. We prove that if $1 \leq q < r \leq \infty$, then $W^{1,(p,q)}(\Omega)$ is strictly included in $W^{1,(p,r)}(\Omega)$.

We also extend the Morrey embedding theorem to the Sobolev-Lorentz spaces $H_0^{1,(p,q)}(\Omega)$ when $n < p < \infty$. Namely, we prove that the Sobolev-Lorentz spaces $H_0^{1,(p,q)}(\Omega)$ embed into the space of Hölder continuous functions on $\overline{\Omega}$ with exponent $1 - \frac{n}{p}$ whenever $\Omega \subset \mathbf{R}^n$ is open, $1 \leq n < p < \infty$ and $1 \leq q \leq \infty$.