
Contents

Preface	xiii
Preface to the Second Edition	xiii
Preface to the First Edition	xv
Acknowledgments	xxi
Second Edition	xxi
First Edition	xxii
Part 1. Functions of One Variable	
Chapter 1. Monotone Functions	3
§1.1. Continuity	3
§1.2. Differentiability	9
Chapter 2. Functions of Bounded Pointwise Variation	29
§2.1. Pointwise Variation	29
§2.2. Continuity	34
§2.3. Differentiability	40
§2.4. Monotone Versus BPV	44
§2.5. The Space $BPV(I; Y)$	47
§2.6. Composition in $BPV(I; Y)$	55
§2.7. Banach Indicatrix	59
Chapter 3. Absolutely Continuous Functions	67
§3.1. $AC(I; Y)$ Versus $BPV(I; Y)$	67
§3.2. The Fundamental Theorem of Calculus	71

§3.3.	Lusin (N) Property	84
§3.4.	Superposition in $AC(I; Y)$	91
§3.5.	Chain Rule	95
§3.6.	Change of Variables	100
§3.7.	Singular Functions	103
Chapter 4. Decreasing Rearrangement		111
§4.1.	Definition and First Properties	111
§4.2.	Function Spaces and Decreasing Rearrangement	126
Chapter 5. Curves		133
§5.1.	Rectifiable Curves	133
§5.2.	Arclength	143
§5.3.	Length Distance	146
§5.4.	Curves and Hausdorff Measure	149
§5.5.	Jordan's Curve Theorem	152
Chapter 6. Lebesgue–Stieltjes Measures		157
§6.1.	Measures Versus Increasing Functions	157
§6.2.	Vector-valued Measures Versus $BPV(I; Y)$	168
§6.3.	Decomposition of Measures	177
Chapter 7. Functions of Bounded Variation and Sobolev Functions		183
§7.1.	$BV(\Omega)$ Versus $BPV(\Omega)$	183
§7.2.	Sobolev Functions Versus Absolutely Continuous Functions	188
§7.3.	Interpolation Inequalities	196
Chapter 8. The Infinite-Dimensional Case		205
§8.1.	The Bochner Integral	205
§8.2.	L^p Spaces on Banach Spaces	212
§8.3.	Functions of Bounded Pointwise Variation	220
§8.4.	Absolute Continuous Functions	224
§8.5.	Sobolev Functions	229
Part 2. Functions of Several Variables		
Chapter 9. Change of Variables and the Divergence Theorem		239
§9.1.	Directional Derivatives and Differentiability	239
§9.2.	Lipschitz Continuous Functions	242
§9.3.	The Area Formula: The C^1 Case	249

§9.4. The Area Formula: The Differentiable Case	262
§9.5. The Divergence Theorem	273
Chapter 10. Distributions	281
§10.1. The Spaces $\mathcal{D}_K(\Omega)$, $\mathcal{D}(\Omega)$, and $\mathcal{D}'(\Omega)$	281
§10.2. Order of a Distribution	288
§10.3. Derivatives of Distributions and Distributions as Derivatives	290
§10.4. Rapidly Decreasing Functions and Tempered Distributions	298
§10.5. Convolutions	302
§10.6. Convolution of Distributions	305
§10.7. Fourier Transforms	309
§10.8. Littlewood–Paley Decomposition	316
Chapter 11. Sobolev Spaces	319
§11.1. Definition and Main Properties	319
§11.2. Density of Smooth Functions	325
§11.3. Absolute Continuity on Lines	336
§11.4. Duals and Weak Convergence	344
§11.5. A Characterization of $W^{1,p}(\Omega)$	349
Chapter 12. Sobolev Spaces: Embeddings	355
§12.1. Embeddings: $mp < N$	356
§12.2. Embeddings: $mp = N$	372
§12.3. Embeddings: $mp > N$	378
§12.4. Superposition	387
§12.5. Interpolation Inequalities in \mathbb{R}^N	399
Chapter 13. Sobolev Spaces: Further Properties	411
§13.1. Extension Domains	411
§13.2. Poincaré Inequalities	430
§13.3. Interpolation Inequalities in Domains	449
Chapter 14. Functions of Bounded Variation	459
§14.1. Definition and Main Properties	459
§14.2. Approximation by Smooth Functions	462
§14.3. Bounded Pointwise Variation on Lines	468
§14.4. Coarea Formula for BV Functions	478
§14.5. Embeddings and Isoperimetric Inequalities	482

§14.6. Density of Smooth Sets	489
§14.7. A Characterization of $BV(\Omega)$	493
Chapter 15. Sobolev Spaces: Symmetrization	497
§15.1. Symmetrization in L^p Spaces	497
§15.2. Lorentz Spaces	502
§15.3. Symmetrization of $W^{1,p}$ and BV Functions	504
§15.4. Sobolev Embeddings Revisited	510
Chapter 16. Interpolation of Banach Spaces	517
§16.1. Interpolation: K -Method	517
§16.2. Interpolation: J -Method	526
§16.3. Duality	530
§16.4. Lorentz Spaces as Interpolation Spaces	535
Chapter 17. Besov Spaces	539
§17.1. Besov Spaces $B_q^{s,p}$	539
§17.2. Some Equivalent Seminorms	545
§17.3. Besov Spaces as Interpolation Spaces	551
§17.4. Sobolev Embeddings	561
§17.5. The Limit of $B_q^{s,p}$ as $s \rightarrow 0^+$ and $s \rightarrow m^-$	565
§17.6. Besov Spaces and Derivatives	571
§17.7. Yet Another Equivalent Norm	578
§17.8. And More Embeddings	585
Chapter 18. Sobolev Spaces: Traces	591
§18.1. The Trace Operator	592
§18.2. Traces of Functions in $W^{1,1}(\Omega)$	598
§18.3. Traces of Functions in $BV(\Omega)$	605
§18.4. Traces of Functions in $W^{1,p}(\Omega)$, $p > 1$	606
§18.5. Traces of Functions in $W^{m,1}(\Omega)$	621
§18.6. Traces of Functions in $W^{m,p}(\Omega)$, $p > 1$	626
§18.7. Besov Spaces and Weighted Sobolev Spaces	626
Appendix A. Functional Analysis	635
§A.1. Topological Spaces	635
§A.2. Metric Spaces	638
§A.3. Topological Vector Spaces	639

§A.4. Normed Spaces	643
§A.5. Weak Topologies	645
§A.6. Hilbert Spaces	648
Appendix B. Measures	651
§B.1. Outer Measures and Measures	651
§B.2. Measurable and Integrable Functions	655
§B.3. Integrals Depending on a Parameter	662
§B.4. Product Spaces	663
§B.5. Radon–Nikodym’s and Lebesgue’s Decomposition Theorems	665
§B.6. Signed Measures	666
§B.7. L^p Spaces	668
§B.8. Modes of Convergence	673
§B.9. Radon Measures	676
§B.10. Covering Theorems in \mathbb{R}^N	678
Appendix C. The Lebesgue and Hausdorff Measures	681
§C.1. The Lebesgue Measure	681
§C.2. The Brunn–Minkowski Inequality	683
§C.3. Mollifiers	687
§C.4. Maximal Functions	694
§C.5. BMO Spaces	695
§C.6. Hardy’s Inequality	698
§C.7. Hausdorff Measures	699
Appendix D. Notes	703
Appendix E. Notation and List of Symbols	711
Bibliography	717
Index	729