

ERRATA OF

A. C. G. Mennucci.

On asymmetric distances.

Analysis and Geometry in Metric Spaces, 1:200–231, 2013.

DOI: 10.2478/agms-2013-0004.

URL <http://cvgmt.sns.it/person/109/>

The following are minor corrections in the paper; there is not (yet) an official errata in the journal.

There is also an improvement for Prop.3.18.

In Lemma 2.17 we should also ask that $\gamma \circ \varphi^{-1} \in C_0$, otherwise Φ may fail to be invertible.

In Theorem 2.19 point 3 is for when $\gamma \in C$.

(This mistake may actually be due to the fact that, in a version of LaTeX that I was using last year, the \notin was rendered as \in).

A point 5 may be added for the case when γ is not in C , in this case

if γ is a "limit point" for C then $\text{Len}^b(\gamma) = \liminf \text{len}(\xi)$ for $\xi \in C, \xi \rightarrow \gamma$

if γ is not a "limit point" for C , $\text{Len}^b(\gamma) = +\infty$.

Lemma 2.24 claims that " φ, φ^{-1} are Lipschitz" but the homeomorphism φ that is defined in the proof is AC and not Lipschitz. Note that φ^{-1} is Lipschitz.

Due to Lemma 2.23, this mistake does not affect the Prop. 2.25

For the same reason, the footnote 12 is to be deleted.

In Lemma 2.24 the homeomorphism φ is increasing (as required by Definition 2.4).

The proof of Proposition 3.18 contains a glitch; it may be the case that the given class C contains curves that have infinite length (this is permitted by the definition of "run-continuous length structure"); in this case obviously C cannot be a subset of C_r . So as a first step in the proof we must ignore all curves of infinite length in C . Since we are interested in the induced distance, this does not affect the result.

Proposition 3.18 can be improved. In point 2 there is no need to assume that the length is lower semi continuous.

Summarizing the above two remarks, the proof of 3.18 changes as follows.

Proof. Let E be the set of $\gamma \in C$ such that $\text{len}(\gamma) < \infty$.

Note that b is also the distance induced by (E, len) .

Consider the class C_r in (M, b) and induce b^r from (C_r, Len^b) .

Recalling from Theorem 2.19 that $\text{len} \geq \text{Len}^b$ on E , we obtain that $E \subseteq C_r$.

Consider the class E in (M, b) and b^l the distance induced by

(E, Len^b) then (by the point 4 in the aforementioned theorem) $b \equiv b^l$.

Since $E \subseteq C_r$ so $b^l \geq b^r$; but also $b \leq b^r$.

(If moreover len is l.s.c. in the DF topology then $\text{len} = \text{Len}^b$ on C .)

This new version of Prop.3.18 directly proves the Prop. 3.19

The "General Metric Space" (Definition 3.4) also has the axiom

$$\forall x, y \in M, b(x, y) = 0 \Rightarrow x = y;$$

(i.e. it is "strongly separated" as defined in [16]).

In example 4.1 the point 2 should be deleted (it is redundant and it points to a non-existent proposition 1.3 that is instead an example); point 7 repeats the same assertion and references the correct proposition.

In example 4.6 the joining of two paths in C_H is not necessarily in C_H , so the pair (C_H, len_5) is not really a length structure; a larger class C_P is needed, that contains all possible joins of paths in C_H (namely, the paths that are continuous and piecewise injective), so (C_P, len_5) is a length structure.

(This change though does not affect the significance of the example).

In the whole of the paper the notation $[a, b]$ was used to define an interval, but this is an abuse of notations since b is also the distance function.

(In [16] I used the interval $[a, c]$)