1. Introduction

Let $s \in (0,1)$ and $N > 2s$. If $A : \mathbb{R}^N \to \mathbb{R}^N$ is a smooth function, the nonlocal operator

$$(−\Delta)^s_A u(x) = c(N,s) \lim_{\varepsilon \to 0} \int_{B_c(x)} \frac{u(x) - e^{i(x-y) \cdot A(\frac{x+y}{2})} u(y)}{|x-y|^{N+2s}} dy, \quad x \in \mathbb{R}^N,$$

has been recently introduced in [6], where the ground state solutions of $(−\Delta)^s_A u + u = |u|^{p-2}u$ in the three dimensional setting have been obtained via concentration compactness arguments. If $A = 0$, then the above operator is consistent with the usual notion of fractional Laplacian. The motivations that led to its introduction are carefully described in [6] and rely essentially on the Lévy-Khintchine formula for the generator of a general Lévy process. We point out that the normalization constant $c(N,s)$ satisfies

$$\lim_{s \to 1} \frac{c(N,s)}{1-s} = \frac{4N\Gamma(N/2)}{2\pi^{N/2}},$$

where $\Gamma$ denotes the Gamma function. For the sake of completeness, we recall that different definitions of nonlocal magnetic operator are viable, see e.g. [8, 9]. All these notions aim to extend the well-know definition of the magnetic Schrödinger operator

$$(\nabla - iA(x))^2 u = −\Delta u + 2iA(x) \cdot \nabla u + |A(x)|^2 u + iu \text{div}A(x),$$

namely the differential of the energy functional

$$\mathcal{E}_A(u) = \int_{\mathbb{R}^N} |\nabla u - iA(x)u|^2 dx,$$

for which we refer the reader to [1,2,11] and the included references. In order to corroborate the justification for the introduction of $(−\Delta)^s_A$, in this note we prove that a well-known formula due to Brezis, Bourgain and Mironescu (see [3, 4, 10]) for the limit of the Gagliardo semi-norm of $H^s(\Omega)$ as $s \nearrow 1$ extends to the magnetic setting. As a consequence, in a suitable sense, from the nonlocal to the local regime, it holds

$$(−\Delta)^s_A u \rightharpoonup (\nabla - iA(x))^2 u,$$

for $s \nearrow 1$.

Precisely, we have the following

**Theorem 1.1** (Magnetic Brezis-Bourgain-Mironescu). Let $\Omega \subset \mathbb{R}^N$ be an open bounded set with Lipschitz boundary and $A \in C^2(\mathbb{R}^N)$. Then, for every $u \in H^1_{A}(\Omega)$, we have

$$\lim_{s \to 1} (1-s) \int_{\Omega} \int_{\Omega} \frac{|u(x) - e^{i(x-y) \cdot A(\frac{x+y}{2})} u(y)|^2}{|x-y|^{N+2s}} dxdy = K_N \int_{\Omega} |\nabla u - iA(x)u|^2 dx,$$

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where
\begin{equation}
K_N = \frac{1}{2} \int_{\mathbb{S}^{N-1}} |\omega \cdot e|^2 d\mathcal{H}^{N-1}(\omega),
\end{equation}
being $\mathbb{S}^{N-1}$ the unit sphere and $e$ any unit vector in $\mathbb{R}^N$.

As a variant of Theorem 1.1, if $H^1_{0, A}(\Omega)$ denotes the closure of $C_0^\infty(\Omega)$ in $H^1_A(\Omega)$, there holds

**Theorem 1.2.** Let $\Omega \subset \mathbb{R}^N$ be an open bounded set with Lipschitz boundary and $A \in C^2(\mathbb{R}^N)$. Then, for every $u \in H^1_{0, A}(\Omega)$, we have
\[ \lim_{s \to 1} (1-s) \int_{\mathbb{R}^N} \frac{|u(x) - e^{i(x - y) \cdot A(\frac{x + y}{2})} u(y)|^2}{|x - y|^{N+2s}} dxdy = K_N \int_{\Omega} |\nabla u - iA(x)u|^2 dx. \]

**Notations.** Let $\Omega \subset \mathbb{R}^N$ be an open set. We denote by $L^2(\Omega, \mathbb{C})$ the Lebesgue space of complex valued functions with summable square. For $s \in (0, 1)$, the magnetic Gagliardo semi-norm is
\[ [u]_{H^s_A(\Omega)} := \sqrt{\int_{\Omega} \int_{\Omega} \frac{|u(x) - e^{i(x - y) \cdot A(\frac{x + y}{2})} u(y)|^2}{|x - y|^{N+2s}} dxdy}. \]

We denote by $H^1_A(\Omega)$ the space of functions $u \in L^2(\Omega, \mathbb{C})$ such that $[u]_{H^1_A(\Omega)} < \infty$ endowed with
\[ \| u \|_{H^1_A(\Omega)} := \sqrt{\| u \|^2_{L^2(\Omega)} + [u]_{H^1_A(\Omega)}^2}. \]

We also consider
\[ [u]_{H^1_A(\Omega)} := \sqrt{\int_{\Omega} |\nabla u - iA(x)u|^2 dx}, \]
and define $H^1_A(\Omega)$ as the space of functions $u \in L^2(\Omega, \mathbb{C})$ such that $[u]_{H^1_A(\Omega)} < \infty$ endowed with
\[ \| u \|_{H^1_A(\Omega)} := \sqrt{\| u \|^2_{L^2(\Omega)} + [u]_{H^1_A(\Omega)}^2}. \]

We denote by $B(x_0, R)$ the ball in $\mathbb{R}^N$ of center $x_0$ and radius $R > 0$. For any set $E \subset \mathbb{R}^N$ we will denote by $E^c$ the complement of $E$. For $A, B \subset \mathbb{R}^N$ open and bounded, $A \Subset B$.

### 2. Preliminary results

We start with the following Lemma.

**Lemma 2.1.** Assume that $A : \mathbb{R}^N \to \mathbb{R}^N$ is locally bounded. Then, for any compact $V \subset \mathbb{R}^N$ with $\Omega \Subset V$, there exists $C = C(A, V) > 0$ such that
\[ \int_{\mathbb{R}^N} |u(y + h) - e^{ih \cdot A(y + \frac{h}{2})} u(y)|^2 dy \leq C|h|^2 \| u \|^2_{H^1_A(\mathbb{R}^N)}, \]
for all $u \in H^1_A(\mathbb{R}^N)$ such that $u = 0$ on $V^c$ and any $h \in \mathbb{R}^N$ with $|h| \leq 1$.

**Proof.** Assume first that $u \in C^\infty_0(\mathbb{R}^N)$ with $u = 0$ on $V^c$. Fix $y, h \in \mathbb{R}^N$ and define
\[ \varphi(t) := e^{i(1-t)h \cdot A(y + \frac{h}{2})} u(y + th), \quad t \in [0, 1]. \]

Then we have
\[ u(y + h) - e^{ih \cdot A(y + \frac{h}{2})} u(y) = \varphi(1) - \varphi(0) = \int_0^1 \varphi'(t) dt, \]
and since
\[ \varphi'(t) = e^{i(1-t)h \cdot A(y + \frac{h}{2})} h \cdot \left( \nabla_y u(y + th) - iA(y + \frac{h}{2}) u(y + th) \right), \]
by Hölder inequality we get
\[ |u(y + h) - e^{ih \cdot A(y + \frac{h}{2})} u(y)|^2 \leq |h|^2 \int_0^1 \left| \nabla_y u(y + th) - iA(y + \frac{h}{2}) u(y + th) \right|^2 dt. \]
Therefore, integrating with respect to $y$ over $\mathbb{R}^N$ and using Fubini’s Theorem, we get

$$
\int_{\mathbb{R}^N} |u(y + h) - e^{ih\cdot A(y + \frac{h}{2})} u(y)|^2 dy \leq |h|^2 \int_0^1 dt \int_{\mathbb{R}^N} \left| \nabla_y u(y + th) - iA(y + \frac{h}{2}) u(y + th) \right|^2 dy
$$

$$
= |h|^2 \int_0^1 dt \int_{\mathbb{R}^N} \left| \nabla_z u(z) - iA(z) u(z) \right|^2 dz
$$

$$
\leq 2|h|^2 \int_{\mathbb{R}^N} |\nabla_z u(z) - iA(z) u(z)|^2 dz
$$

$$
+ 2|h|^2 \int_V \left| \left( z + \frac{1}{2} \right) - A(z) \right|^2 |u(z)|^2 dz.
$$

Then, since $A$ is bounded on the set $V$, we have for some constant $C > 0$

$$
\int_{\mathbb{R}^N} |u(y + h) - e^{ih\cdot A(y + \frac{h}{2})} u(y)|^2 dy \leq C|h|^2 \left( \int_{\mathbb{R}^N} |\nabla_z u(z) - iA(z) u(z)|^2 dz + \int_{\mathbb{R}^N} |u(z)|^2 dz \right)
$$

$$
= C|h|^2 \|u\|^2_{H_1^A(\mathbb{R}^N)}.\tag{2.1}
$$

When dealing with a general $u$ we can argue by a density argument. \hfill \square

**Lemma 2.2.** Let $\Omega \subset \mathbb{R}^N$ be an open bounded set with Lipschitz boundary, $V \subset \mathbb{R}^N$ a compact set with $\Omega \Subset V$ and $A : \mathbb{R}^N \to \mathbb{R}^N$ locally bounded. Then there exists $C(\Omega, V, A) > 0$ such that for any $u \in H_1^A(\Omega)$ there exists $Eu \in H_1^A(\mathbb{R}^N)$ such that $Eu = u$ in $\Omega$, $Eu = 0$ in $V^c$ and

$$
\|Eu\|_{H_1^A(\mathbb{R}^N)} \leq C(\Omega, V, A)\|u\|_{H_1^A(\Omega)}.
$$

**Proof.** Observe that, for any bounded set $W \subset \mathbb{R}^N$ there exist $C_1(A, W), C_2(A, W) > 0$ with

$$
C_1(A, W)\|u\|_{H_1^A(W)} \leq \|u\|_{H_1^A(\mathbb{R}^N)} \leq C_2(A, W)\|u\|_{H_1^A(W)}, \quad \text{for any } u \in H_1^A(W).
$$

This follows easily, via simple computations, by the definition of the norm of $H_1^A(W)$ and in view of the local boundedness assumption on the potential $A$, see [1, Lemma 2.3] for further details. Now, by the standard extension property for $H_1^A(\Omega)$ (see e.g. [7, Theorem 1, p.254]) there exists $C(\Omega, V) > 0$ such that for any $u \in H_1^A(\Omega)$ there exists a function $Eu \in H_1^A(\mathbb{R}^N)$ such that $Eu = u$ in $\Omega$, $Eu = 0$ in $V^c$ and $\|Eu\|_{H_1^A(\mathbb{R}^N)} \leq C(\Omega, V)\|u\|_{H_1^A(\Omega)}$. Then, for any $u \in H_1^A(\Omega)$, we get

$$
\|Eu\|_{H_1^A(\mathbb{R}^N)} = \|Eu\|_{H_1^A(V)} \leq C_2(A, V)\|Eu\|_{H_1^A(V)} = C_2(A, V)\|Eu\|_{H_1^A(\mathbb{R}^N)}
$$

$$
\leq C(\Omega, V)C_2(A, V)\|u\|_{H_1^A(\Omega)} \leq C(\Omega, V)C_2(A, V)C_1^{-1}(A, \Omega)\|u\|_{H_1^A(\Omega)},
$$

which concludes the proof. \hfill \square

We can now prove the following result:

**Lemma 2.3.** Let $A : \mathbb{R}^N \to \mathbb{R}^N$ be locally bounded. Let $u \in H_1^A(\Omega)$ and $\rho \in L_1(\mathbb{R}^N)$ with $\rho \geq 0$. Then

$$
\int_\Omega \int_\Omega \frac{|u(x) - e^{i(x-y)\cdot A\left(\frac{x+y}{2}\right)} u(y)|^2}{|x-y|^2} \rho(x-y) \ dx dy \leq C\|\rho\|_{L^1} \|u\|^2_{H_1^A(\Omega)}
$$

where $C$ depends only on $\Omega$ and $A$.

**Proof.** Let $V \subset \mathbb{R}^N$ be a fixed compact set with $\Omega \Subset V$. Given $u \in H_1^A(\Omega)$, by Lemma 2.2, there exists a function $\tilde{u} \in H_1^A(\mathbb{R}^N)$ with $\tilde{u} = u$ on $\Omega$ and $\tilde{u} = 0$ on $V^c$. By Lemma 2.1 and 2.2,

$$
(2.1) \quad \int_{\mathbb{R}^N} |\tilde{u}(y + h) - e^{ih\cdot A(y + \frac{h}{2})} \tilde{u}(y)|^2 dy \leq C|h|^2 \|\tilde{u}\|^2_{H_1^A(\mathbb{R}^N)} \leq C|h|^2 \|u\|^2_{H_1^A(\Omega)}.
$$
for some positive constant $C$ depending on $\Omega$ and $A$. Then, in light of (2.1), we get

$$
\int_{\Omega} \int_{\Omega} \frac{|u(x) - e^{i(x-y) \cdot A(\frac{\cdot x}{r})} u(y)|^2}{|x-y|^2} dxdy \leq \int_{\mathbb{R}^N} \int_{\mathbb{R}^N} \rho(h) \frac{|\tilde{u}(y) - e^{iA(y+\frac{\cdot y}{r})} \tilde{u}(y)|^2}{|h|^2} dydh
$$

$$
= \int_{\mathbb{R}^N} \rho(h) \left( \int_{\mathbb{R}^N} |\tilde{u}(y) - e^{iA(y+\frac{\cdot y}{r})} \tilde{u}(y)|^2 dy \right) dh
$$

$$
\leq C||\rho||_{L^1} ||u||_{H^s_1(\Omega)}^2,
$$

which concludes the proof.

\begin{proof}
Let $A : \mathbb{R}^N \to \mathbb{R}^N$ be locally bounded and let $u \in H^{1}_{\Lambda}(\Omega)$. Then, we have

$$
(1-s) \int_{\mathbb{R}^{2N}} \frac{|u(x) - e^{i(x-y) \cdot A(\frac{\cdot x}{r})} u(y)|^2}{|x-y|^{N+2s}} dxdy \leq C||u||_{H^s_1(\Omega)}^2
$$

where $C$ depends only on $\Omega$ and $A$.

\begin{proof}
Given $u \in C_c^\infty(\Omega)$, by Lemma 2.1 we have

$$
\int_{\mathbb{R}^N} |u(y) - e^{iA(y+\frac{\cdot y}{r})} u(y)|^2 dy \leq C||h||_{H^s_1(\Omega)}^2
$$

for some $C > 0$ depending on $\Omega$ and $A$ and all $h \in \mathbb{R}^N$ with $|h| \leq 1$. Then, we get

$$
(1-s) \int_{\mathbb{R}^{2N}} \frac{|u(x) - e^{i(x-y) \cdot A(\frac{\cdot x}{r})} u(y)|^2}{|x-y|^{N+2s}} dxdy \leq (1-s) \int_{\mathbb{R}^{2N}} \frac{|u(y) - e^{iA(y+\frac{\cdot y}{r})} u(y)|^2}{|h|^{N+2s}} dydh
$$

$$
= (1-s) \int_{\{h| \leq 1\}} \frac{1}{|h|^{N+2s}} \left( \int_{\mathbb{R}^N} |u(y) - e^{iA(y+\frac{\cdot y}{r})} u(y)|^2 dy \right) dh
$$

$$
+ 4(1-s) \int_{\{|h| \geq 1\}} \frac{1}{|h|^{N+2s}} dh ||u||_{L^2(\Omega)}^2
$$

$$
\leq (1-s) \int_{\{h| \leq 1\}} \frac{1}{|h|^{N+2s-2}} dh ||u||_{H^s_1(\Omega)}^2 + C||u||_{L^2(\Omega)}^2 \leq C||u||_{H^s_1(\Omega)}^2.
$$

The assertion then follows by a density argument.

Assuming now that $A \in C^2(\mathbb{R}^N)$, we are able to give the following result.

**Theorem 2.5.** Assume that $A \in C^2(\mathbb{R}^N)$. Let $u \in H^{1}_{\Lambda}(\Omega)$ and consider a sequence $\{\rho_n\}_{n \in \mathbb{N}}$ of nonnegative radial functions in $L^1(\mathbb{R}^N)$ with

$$
\lim_{n \to \infty} \int_0^\infty \rho_n(r) r^{N-1} dr = 1,
$$

and such that, for every $\delta > 0$,

$$
\lim_{n \to \infty} \int_{\delta}^\infty \rho_n(r) r^{N-1} dr = \lim_{n \to \infty} \int_0^\delta \rho_n(r) r^{N-1} dr = \lim_{n \to \infty} \int_0^\delta \rho_n(r) r^{N+1} dr = 0.
$$

Then, we have

$$
\lim_{n \to \infty} \int_{\Omega} \int_{\Omega} \frac{|u(x) - e^{i(x-y) \cdot A(\frac{\cdot x}{r})} u(y)|^2}{|x-y|^2} \rho_n(x-y) dxdy = 2K_N \int_{\Omega} |\nabla u - iA(x) u|^2 dx
$$

being $K_N$ the constant introduced in (1.1).

\begin{proof}
We follow the main lines of the proof in [3]. Setting

$$
F_n^u(x, y) := \frac{u(x) - e^{i(x-y) \cdot A(\frac{\cdot x}{r})} u(y)}{|x-y|^{1/2}} \rho_n^{1/2}(x-y), \quad x, y \in \Omega, \ n \in \mathbb{N},
$$

by virtue of Lemma 2.3, for all $u, v \in H^1_\Lambda(\Omega)$, recalling (2.2) we have

$$
||F_n^u||_{L^2(\Omega \times \Omega)} - ||F_n^v||_{L^2(\Omega \times \Omega)} \leq ||F_n^u - F_n^v||_{L^2(\Omega \times \Omega)} \leq C||u - v||_{H^1_\Lambda(\Omega)}.
$$
for some $C > 0$ depending on $\Omega$ and $A$. This allows to prove (2.4) for $u \in C^2(\overline{\Omega})$. If we set

$$\varphi(y) := e^{i(x-y) \cdot A(x,y)} u(y),$$

since

$$\nabla_y \varphi(y) = e^{i(x-y) \cdot A(x,y)} \left( \nabla_y u(y) - iA \left( \frac{x+y}{2} \right) u(y) + \frac{i}{2} u(y)(x-y) \cdot \nabla_y A \left( \frac{x+y}{2} \right) \right),$$

if $x \in \Omega$, a second order Taylor expansion gives (since $u, A \in C^2$, then $\nabla_y^2 \varphi$ is bounded on $\overline{\Omega}$)

$$u(x) - e^{i(x-y) \cdot A(x,y)} u(y) = \varphi(x) - \varphi(y) = (\nabla u(x) - iA(x)u(x)) \cdot (x-y) + O(|x-y|^2).$$

Hence, for any fixed $x \in \Omega$,

$$\left| u(x) - e^{i(x-y) \cdot A(x,y)} u(y) \right| \leq \left| (\nabla u(x) - iA(x)u(x)) \cdot \frac{x-y}{|x-y|} \right| + O(|x-y|). \tag{2.5}$$

Fix $x \in \Omega$. If we set $R_x := \text{dist}(x, \partial \Omega)$, integrating with respect to $y$, we have

$$\int_{\Omega} \frac{|u(x) - e^{i(x-y) \cdot A(x,y)} u(y)|^2}{|x-y|^2} \rho_n(x-y) \, dy = \int_{B(x, R_x)} \frac{|u(x) - e^{i(x-y) \cdot A(x,y)} u(y)|^2}{|x-y|^2} \rho_n(x-y) \, dy + \int_{\Omega \setminus B(x, R_x)} \frac{|u(x) - e^{i(x-y) \cdot A(x,y)} u(y)|^2}{|x-y|^2} \rho_n(x-y) \, dy. \tag{2.6}$$

The second integral goes to zero by the first limit of conditions (2.3), since

$$\lim_{n \to \infty} \int_{\Omega \setminus B(x, R_x)} \frac{|u(x) - e^{i(x-y) \cdot A(x,y)} u(y)|^2}{|x-y|^2} \rho_n(x-y) \, dy \leq C \lim_{n \to \infty} \int_{B^c(0, R_x)} \rho_n(z) \, dz = 0.$$

Now, in light of (2.5), following [3] we compute

$$\int_{B(x, R_x)} \frac{|u(x) - e^{i(x-y) \cdot A(x,y)} u(y)|^2}{|x-y|^2} \rho_n(x-y) \, dy = Q_N |\nabla u(x) - iA(x)u(x)|^2 \int_0^{R_x} r^{N-1} \rho_n(r) \, dr + O \left( \int_0^{R_x} r^N \rho_n(r) \, dr \right) + O \left( \int_0^{R_x} r^{N+1} \rho_n(r) \, dr \right),$$

where we have set

$$Q_N = \int_{S^{N-1}} |\omega \cdot e|^2 \, d\mathcal{H}^{N-1}(\omega),$$

being $e \in \mathbb{R}^N$ a unit vector. Letting $n \to \infty$ in (2.6), the result follows by dominated convergence. \(\square\)

3. PROOFS OF THEOREM 1.1 AND 1.2

3.1. Proof of Theorem 1.1. If $r_\Omega := \text{diam}(\Omega)$, we consider a radial cut-off $\psi \in C^\infty_c(\mathbb{R}^N)$, $\psi(x) = \psi_0(|x|)$ with $\psi_0(t) = 1$ for $t < r_\Omega$ and $\psi_0(t) = 0$ for $t > 2r_\Omega$. Then, by construction, $\psi_0(|x-y|) = 1$, for every $x, y \in \Omega$. Furthermore, let $\{s_n\}_{n \in \mathbb{N}} \subset (0, 1)$ be a sequence with $s_n \not\to 1$ as $n \to \infty$ and consider the sequence of radial functions in $L^1(\mathbb{R}^N)$

$$\rho_n(|x|) = \frac{2(1 - s_n)}{|x|^{N + 2s_n - 2}} \psi_0(|x|), \quad x \in \mathbb{R}^N, \quad n \in \mathbb{N}. \tag{3.1}$$

Notice that (2.2) holds, since

$$\lim_{n \to \infty} \int_0^{r_\Omega} \rho_n(r) r^{N-1} \, dr = \lim_{n \to \infty} 2(1 - s_n) \int_0^{r_\Omega} \frac{1}{r^{2s_n-1}} \, dr = \lim_{n \to \infty} \frac{2^{2s_n} - 2}{r_\Omega^{2s_n-1}} = 1,$$

and

$$\lim_{n \to \infty} \int_{r_\Omega}^{2r_\Omega} \rho_n(r) r^{N-1} \, dr = \lim_{n \to \infty} 2(1 - s_n) \int_{r_\Omega}^{2r_\Omega} \frac{\psi_0(r)}{t^{2s_n-1}} \, dt = 0.$$
In a similar fashion, for any \( \delta > 0 \), there holds
\[
\lim_{n \to \infty} \int_\delta^\infty \rho_n(r)r^{N-1}dr \leq \lim_{n \to \infty} 2(1-s_n) \int_{\delta}^{2^r_\infty} \frac{1}{t^{2s_n-1}}dt = 0,
\]
\[
\lim_{n \to \infty} \int_0^\delta \rho_n(r)r^Ndr \leq \lim_{n \to \infty} 2(1-s_n) \int_0^\delta \frac{1}{t^{2s_n-2}}dt = 0,
\]
\[
\lim_{n \to \infty} \int_0^\delta \rho_n(r)r^{N+1}dr \leq \lim_{n \to \infty} 2(1-s_n) \int_0^\delta \frac{1}{t^{2s_n-3}}dt = 0.
\]
Then Theorem 1.1 follows directly from Theorem 2.5 using \( \rho_n \) as defined in (3.1).

3.2. Proof of Theorem 1.2. In light of Theorem 1.1 and since \( u = 0 \) on \( \Omega^c \), we have
\[
\lim_{s \nearrow 1} (1-s) \int_{\mathbb{R}^N} \frac{|u(x) - e^{i(x-y) \cdot A(x)} u(y)|^2}{|x-y|^{N+2s}} dxdy = K_N \int_\Omega |\nabla u - iA(x)u|^2 dx + \lim_{s \nearrow 1} R_s,
\]
where we have set
\[
R_s := 2(1-s) \int_\Omega \int_{\mathbb{R}^N \setminus \Omega} \frac{|u(x)|^2}{|x-y|^{N+2s}} dxdy.
\]
On the other hand, arguing as in the proof of [5, Proposition 2.8], we get \( R_s \to 0 \) as \( s \nearrow 1 \) when \( u \in C^\infty_c(\Omega) \) and, on account of Lemma 2.4, for general function in \( \mathcal{H}^{s,p}_0(\Omega) \) by a density argument.

References


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