SUPPLEMENTARY MATERIAL TO SECOND-ORDER EDGE-PENALIZATION IN THE AMBROSIO-TORTORELLI FUNCTIONAL

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1. Convergence Behaviour of Alternating Minimization Algorithms

The convergence indicator e^k is plotted vs. the number of iterations in several examples in Figure 1 and 2. Figure 3 illustrates two cases of parameters where alternating minimization on the Ambrosio-Tortorelli functional did not converge.

2. One-dimensional Structure

Figure 4 displays further results for the one-dimensional structure with the parameters $\alpha = 10^{-2}$, $\gamma = 10^{-3}$, $\varepsilon = 9 * 10^{-2}$.

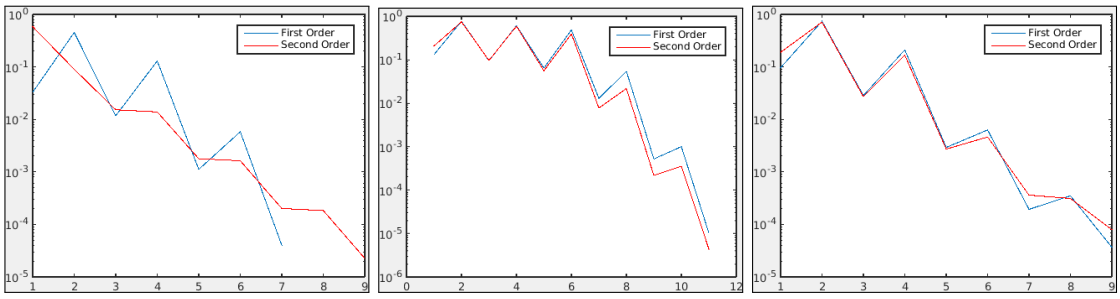

FIGURE 1. Convergence history of e^k vs. number of iterations k. One-dimensional example (left, $\alpha = 10^{-2}$, $\gamma = 10^{-3}$, $\varepsilon = 9 * 10^{-2}$), ellipse (middle, $\alpha = 10^{-2}$, $\gamma = 10^{-3}$, $\varepsilon = 3 * 10^{-2}$), two circles (right, $\alpha = 10^{-2}$, $\gamma = 10^{-3}$, $\varepsilon = 3 * 10^{-2}$).

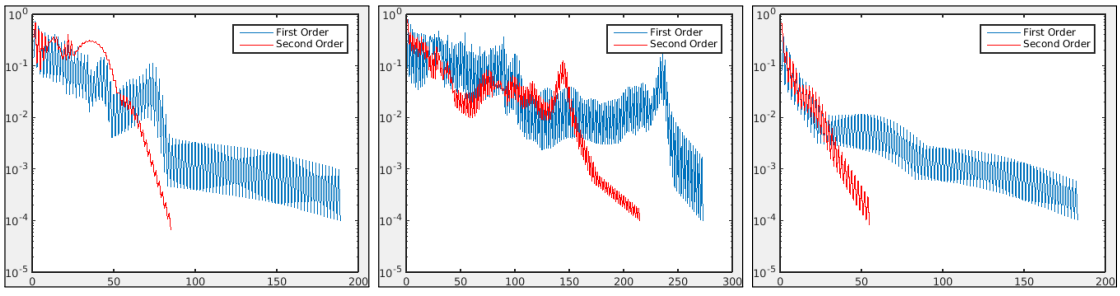

FIGURE 2. Convergence history of e^k vs. number of iterations k. Kodak image 2 (left, $\alpha = 10^{-2}$, $\gamma = 10^{-3}$, $\varepsilon = 3 * 10^{-2}$), Kodak image 7 (middle, $\alpha = 10^{-2}$, $\gamma = 10^{-3}$, $\varepsilon = 7 * 10^{-2}$), Kodak image 23 (right, $\alpha = 10^{-2}$, $\gamma = 10^{-3}$, $\varepsilon = 7 * 10^{-2}$).

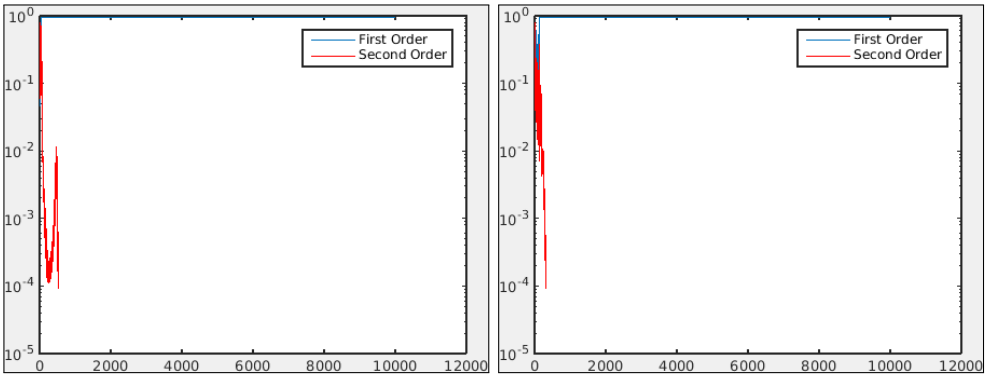

FIGURE 3. Convergence history of e^k vs. number of iterations k. Kodak image 7 (left, $\alpha = 10^{-2}$, $\gamma = 7{*}10^{-4}$, $\varepsilon = 7{*}10^{-2}$), Kodak image 23 (right, $\alpha = 10^{-2}$, $\gamma = 7{*}10^{-4}$, $\varepsilon = 7 * 10^{-2}$).

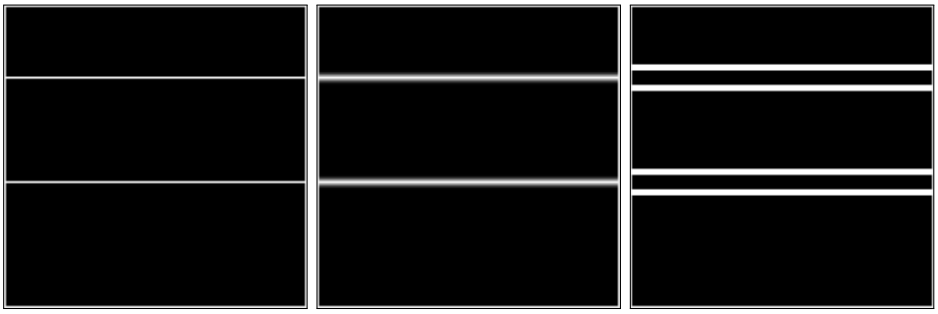

FIGURE 4. One-dimensional structure (from left to right): Image g, resulting v in the Ambrosio-Tortorelli model, resulting v in the second-order model, binary plot of the level set $\{v > 1.005\}$ in the second-order model, $\varepsilon = 9 * 10^{-2}$.

3. Results on Kodak Images

We display some examples of results on the Kodak images 2, 7, and 23, displayed in Figure 5. Figures 6, 7, 8 display the resulting segmentation v for the Kodak image 2 with $\alpha = 10^{-2}$ and different values of γ and ε. Figures 9 and 10 display the results for the Kodak image 7 with $\varepsilon = 7{*}10^{-2}$ and different values of α and γ. Figure 11 displays the resulting v in Kodak image 23 for $\alpha = 10^{-2}$, $\gamma = 10^{-3}$, $\varepsilon = 7 * 10^{-2}$.

4. Reconstructed Images

Figures 12, 13, 14, and 15 display the resulting u in the different models with the parameter settings in the paper. Figures 16, 17, 18 display results for the Kodak image nr 2 with $\alpha = 10^{-2}$ and different values of γ and ε. Figures 19 and 20 display the results for the Kodak image nr 7 with $\varepsilon = 7 * 10^{-2}$ and different values of α and γ. Figure 21 displays the resulting u in Kodak image nr 23 for both models with parameters $\alpha = 10^{-2}, \gamma = 10^{-3}, \varepsilon = 7 * 10^{-2}$.

FIGURE 5. Kodak image 2 (left), Kodak image 7 (middle), Kodak image 23 (right).

FIGURE 6. Kodak image 2: resulting v in the Ambrosio-Tortorelli model (left) and in the second order model (right), $\gamma = 10^{-3}$, $\varepsilon = 3 * 10^{-2}$.

FIGURE 7. Kodak image 2: resulting v in the Ambrosio-Tortorelli model (left) and in the second order model (right), $\gamma = 7 * 10^{-3}$, $\varepsilon = 6 * 10^{-2}$.

FIGURE 8. Kodak image 2: resulting u in the Ambrosio-Tortorelli model (left) and in the second order model (right), $\gamma = 7 * 10^{-4}$, $\varepsilon = 6 * 10^{-2}$.

FIGURE 9. Kodak image 7: resulting v in the Ambrosio-Tortorelli model (left) and in the second order model (right), both with $\alpha = 10^{-2}$, $\gamma = 7 * 10^{-3}$.

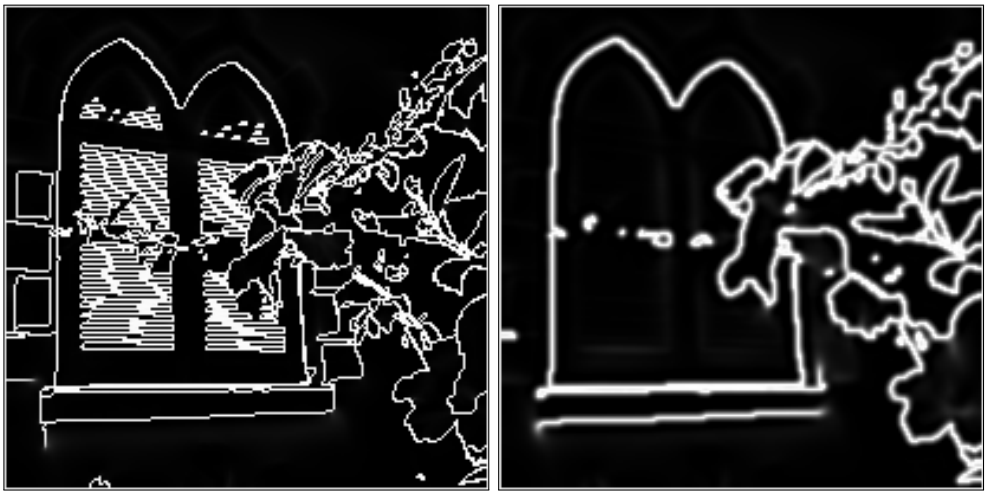

FIGURE 10. Kodak image 7: resulting v in the Ambrosio-Tortorelli model (left) and in the second order model (right), both with $\alpha = 7 * 10^{-2}$, $\gamma = 10^{-3}$.

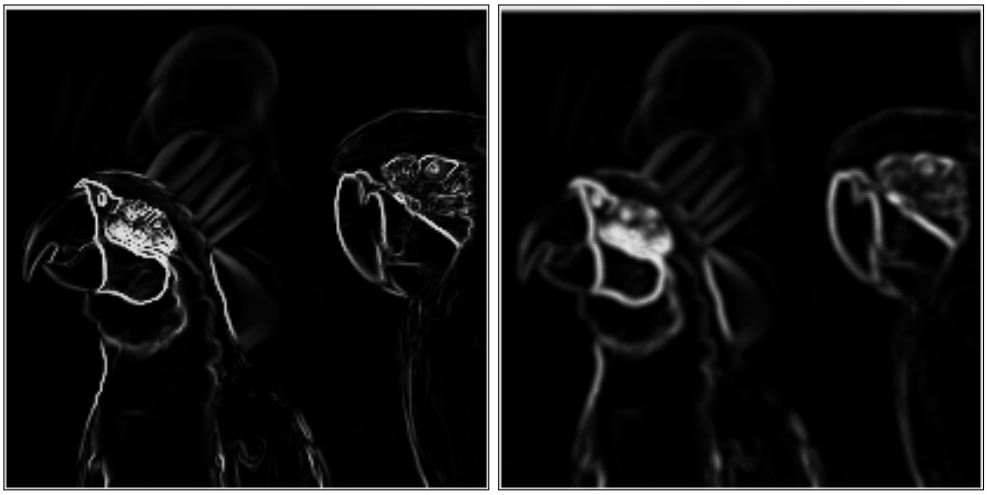

FIGURE 11. Kodak image 23: resulting v in the Ambrosio-Tortorelli model (left) and in the second order model (right).

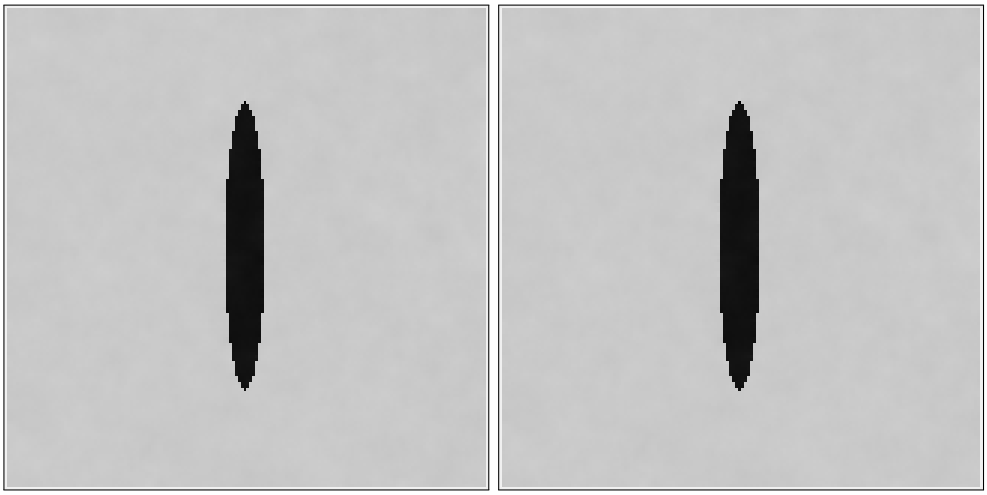

FIGURE 12. Ellipse: resulting u in the Ambrosio-Tortorelli model (left) and in the second order model (right).

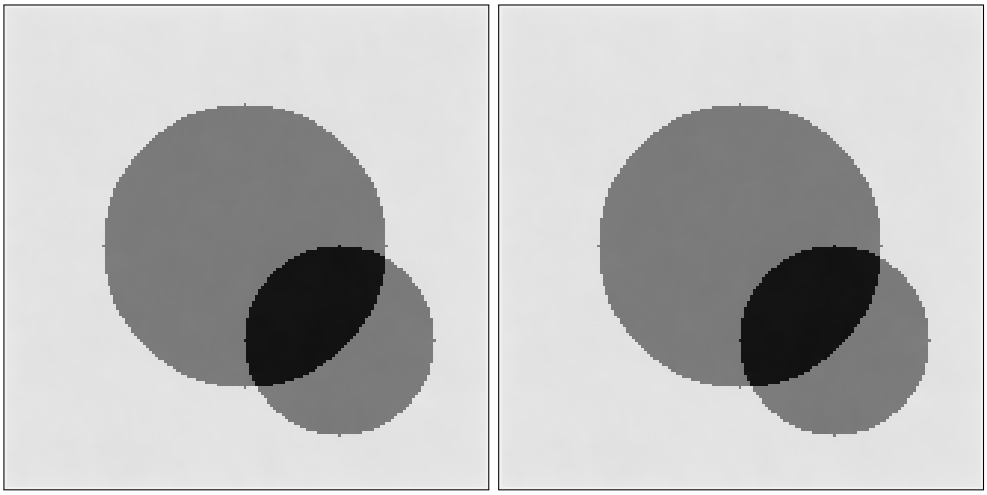

FIGURE 13. Two circles: resulting u in the Ambrosio-Tortorelli model (left) and in the second order model (right).

FIGURE 14. Sisse image: resulting u in the Ambrosio-Tortorelli model (left) and in the second order model (right).

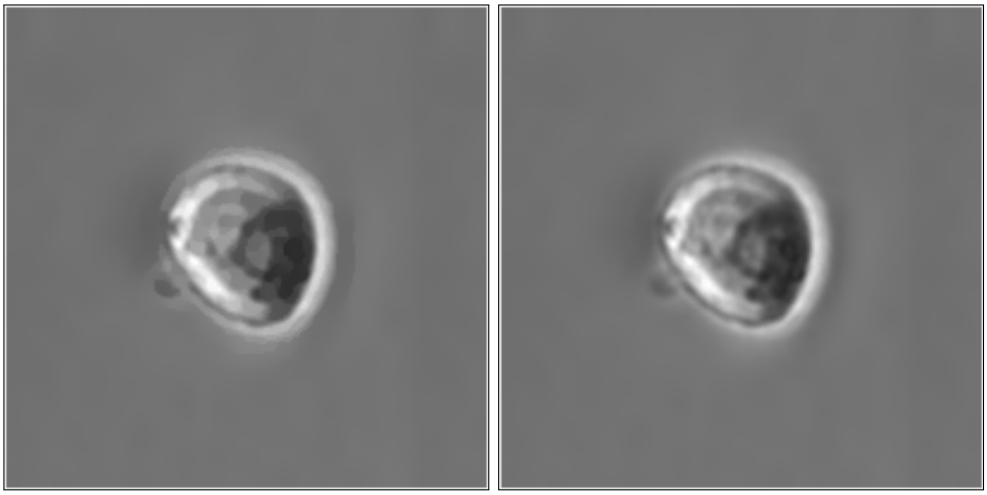

FIGURE 15. Mitosis image: resulting u in the Ambrosio-Tortorelli model (left) and in the second order model (right).

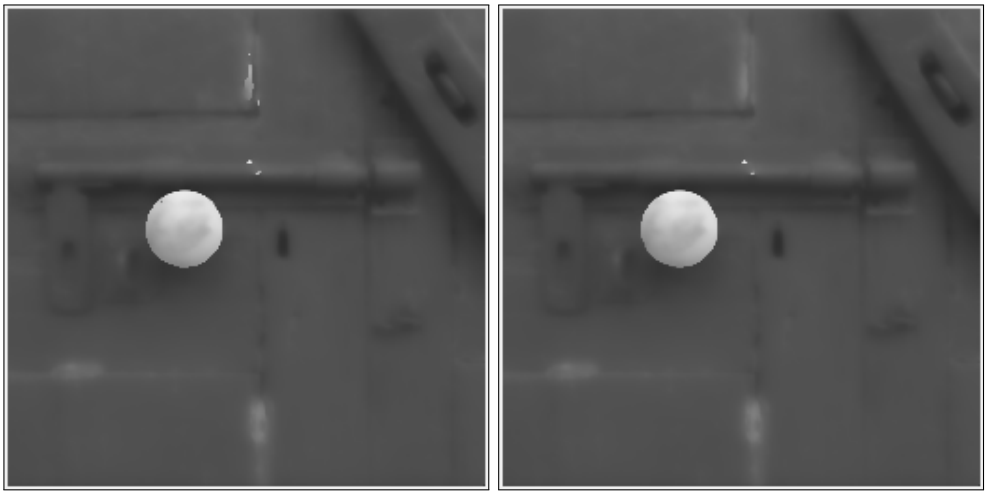

FIGURE 16. Kodak image 2: resulting u in the Ambrosio-Tortorelli model (left) and in the second order model (right), $\gamma = 10^{-3}, \varepsilon = 3 * 10^{-2}$.

FIGURE 17. Kodak image 2: resulting u in the Ambrosio-Tortorelli model (left) and in the second order model (right), $\gamma = 7 * 10^{-3}, \varepsilon = 6 * 10^{-2}$.

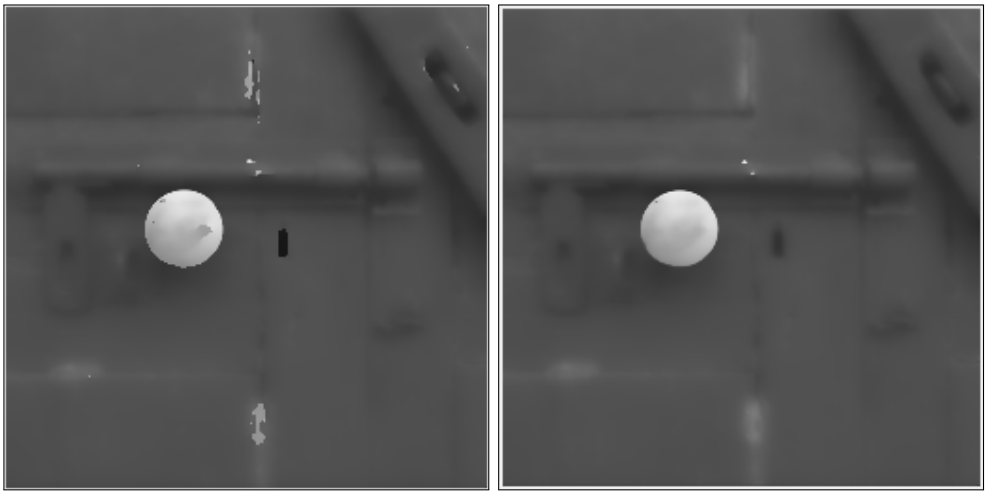

FIGURE 18. Kodak image 2: resulting u in the Ambrosio-Tortorelli model (left) and in the second order model (right), $\gamma = 7 * 10^{-4}, \varepsilon = 6 * 10^{-2}$.

FIGURE 19. Kodak image 7: Resulting u in the Ambrosio-Tortorelli model (left) and in the second order model (right), both with $\alpha = 10^{-2}, \gamma = 7 * 10^{-3}$.

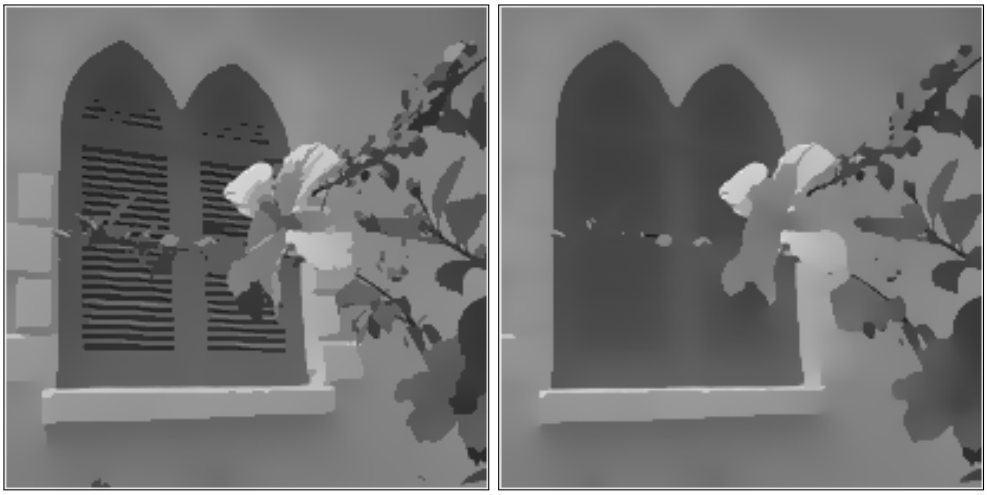

FIGURE 20. Kodak image 7: Resulting u in the Ambrosio-Tortorelli model (left) and in the second order model (right), both with $\alpha = 7 * 10^{-2}, \gamma = 7 * 10^{-3}$.

FIGURE 21. Kodak image 23: resulting u in the Ambrosio-Tortorelli model (left) and in the second order model (right).