CV of LUCA MARTINAZZI

Contacts

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Personal Data

Place and date of birth	Torino, Italy. June 5, 1981
Citizenship	Italian
Marital status	Unmarried
Spoken languages	English (fluent), Italian (fluent), German (fluent), French (basic)

Degrees

M.Sc. in Mathematics	**Scuola Normale Superiore, Pisa, 05/2004**
Grade	110/110 Cum Laude
Title of the Master Thesis	*The non-parametric Plateau problem in arbitrary codimension*
Advisor	Prof. M. Giaquinta
Ph.D. in Mathematics	**ETH Zurich, 03/2009**
Title of the Ph.D. Thesis	*Concentration-Compactness phenomena in conformal geometry*
Advisor	Prof. M. Struwe

Jobs

04/2009 – 09/2009	Postdoc at D-Math, ETH Zurich
10/2009 – 08/2011	Junior visitor at CRM De Giorgi, Pisa
	(1 year is supported by the Swiss National Foundation)
Since 09/2011	Hill Assistant Professor at Rutgers University (NJ)

Education

10/2000 - 09/2004	**Scuola Normale Superiore, Pisa**	Undergraduate student
09/2004 - 08/2005	**Stanford University, CA**	Graduate student
10/2005 - 03/2009	**ETH Zurich**	Graduate student

RESEARCH

My research fields are **Partial Differential Equations, Geometric Analysis** and **Calculus of Variations**.

— Liouville's equation, Q-curvature and Paneitz operators

In the last years I've investigated the higher-order Liouville equation

$$(-\Delta)^m u = V e^{2mu} \quad \text{in } \mathbb{R}^{2m}, \quad m \in \mathbb{N}, \quad V \in \mathbb{R} \tag{1}$$

classifying its many solutions, both in the case $V \geq 0$ and $V < 0$. Improving upon several previous works in this area, I applied the results found for Equation (1) to the problem of prescribing the **Q-curvature** on a closed manifold, or on an open domain in $\mathbb{R}^{2m}$. This corresponds to studying the non-linear equation

$$P_g^{2m} u + Q_g = \bar{Q} e^{2mu} \tag{2}$$

on a Riemannian manifold (M, g) of dimension $2m$. Here Q_g is the Q-curvature of (M, g), P_g^{2m} is the Paneitz operator of (M, g) (roughly speaking a relative of $(-\Delta_g)^m$ which transforms well under conformal changes of the metric g) and $\bar{Q} \in C^\infty(M)$ is the prescribed Q-curvature, in the sense that the metric $e^{2u} g$ has Q-curvature $\bar{Q}$. Equation (2) has much in common with the **Yamabe equation**, and its properties are still under intensive investigations. Surprising results have already appeared as a consequence of the (arbitrarily) **high order of the equation**, which is also a source of interest, together with its subtle **lack of compactness**.

— Adams-Moser-Trudinger embedding

Another focus of my research is the Adams-Moser-Trudinger inequality

$$\sup_{\|u\|^2_{H_0^m(\Omega)} \leq \Lambda_1} \int_\Omega e^{mu^2} dx \leq C|\Omega|$$

for every subset $\Omega \subset \mathbb{R}^{2m}$. Here Λ_1 is a dimensional constant. Critical points of the Adams-Moser-Trudinger embedding solve the equation

$$(-\Delta)^m u = \lambda u e^{u^2} \text{ in } \Omega, \quad u = \partial_\nu u = \ldots = \partial_\nu^{m-1} u = 0 \text{ on } \partial\Omega. \tag{3}$$

Together with M. Struwe, and improving on previous results by O. Druet, F. Robert and M. Struwe, I proved that a sequence (u_k) of solution to (3) with $\lambda = \lambda_k \downarrow 0$ and with $\limsup_{k\to\infty} \|u_k\|_{H_0^m} < \infty$ is either precompact or vanishes asymptotically (up to a subsequence) outside a finite blow-up set. Moreover in the latter case we have a quantization result: up to a subsequence

$$\|u_k\|^2_{H_0^m} = \int_\Omega \lambda_k u_k^2 e^{u_k^2} dx \to L\Lambda_1, \text{ as } k \to \infty \text{ for some } L \in \mathbb{N}.$$

Interestingly this result heavily relies on the understanding of the solutions to (1), which appear naturally after a blow-up procedure.
The existence of critical points of the functional $\int_\Omega e^{mu^2} dx$ under the constraint $\|u\|^2_{H_0^m} = \Lambda > \Lambda_1$ has also been vastly investigated and is part of my research interests.

— **Fractional perimeters and nonlocal minimal surfaces**

In a joint project with L. Ambrosio and G. De Philippis I have started to study the fractional perimeter functional

$$\mathcal{J}_s(E,\Omega) := \mathcal{J}_s^1(E,\Omega) + \mathcal{J}_s^2(E,\Omega),$$

where

$$\mathcal{J}_s^1(E,\Omega) := \int_{E\cap\Omega}\int_{E^c\cap\Omega} \frac{1}{|x-y|^{n+s}}dxdy,$$
$$\mathcal{J}_s^2(E,\Omega) := \int_{E\cap\Omega}\int_{E^c\cap\Omega^c} \frac{1}{|x-y|^{n+s}}dxdy + \int_{E\cap\Omega^c}\int_{E^c\cap\Omega} \frac{1}{|x-y|^{n+s}}dxdy,$$

for a measurable set $E \subset \mathbb{R}^n$, $n \geq 1$, $0 < s < 1$, and a connected open set $\Omega \Subset \mathbb{R}^n$. The functional $\mathcal{J}_s(E,\Omega)$ can be thought of as a fractional perimeter of E in Ω which is non-local in the sense that it is not determined by the behaviour of E in a neighbourhood of $\partial E \cap \Omega$, and which can be finite even if the Hausdorff dimension of ∂E is $n - s > n - 1$. Notice that the term $\mathcal{J}_s^1(E,\Omega)$ is simply half of the fractional Sobolev space seminorm $|\chi_E|_{W^{s,1}(\Omega)}$, where χ_E denotes the characteristic function of E. Roughly speaking this term represents the $(n-s)$-dimensional fractional perimeter of E *inside* Ω, while $\mathcal{J}_s^2$ is the contribution near $\partial\Omega$. This can be made precise when letting $s \uparrow 1$.
In a recent work we proved that

$$\Gamma - \lim_{s\uparrow 1}(1-s)\mathcal{J}_s(E,\Omega) = \omega_{n-1}P(E,\Omega),$$

(here $P(E,\Omega)$ denotes the perimeter of E in Ω in the sense of De Giorgi) and we studied the convergence of local minimizers of $\mathcal{J}_s$ to minimizers of the perimeter.

— **Harmonic maps from B^3 into S^2 minimizing the relaxed energy**

Following the works (among others) of Giaquinta, Modica and Souček, of Bethuel, Brezis and Coron and of Hardt, Lin and Poon, I have been interested in understanding the behaviour of the minimizer of the energy

$$F(T) := \int_{B^3} |\nabla u|^2 dx + 4\pi \mathbf{M}(L), \quad T = [\![\mathrm{Graph}(u)]\!] + L \times [\![S^2]\!],$$

where $u \in W^{1,2}(B^3, S^2)$ and L is a 1-dimensional rectifiable current such that

$$\partial\, [\![\mathrm{Graph}(u)]\!] = -\partial L \times [\![S^2]\!] \quad \text{in } B^3 \times S^2.$$

Currents T as above are called Cartesian currents. The functional F is obtained by relaxing the standard Dirichlet energy. In a symmetric setting I have constructed a minimizer of F having at the same time vertical part ($L \neq 0$) and non-constant graph ($u \not\equiv$ *const*). This was unknown before and casts several interesting open questions, also very related to the regularity of minimal graphs (seen as Cartesian currents) in codimension greater than 1.

Publications

[1] *The non-parametric problem of Plateau in arbitrary codimension - Master thesis*, (2004) available at `http://www.arXiv.org/pdf/math.AP/0411589`

[2] (With M. Giaquinta) *An introduction to the regularity theory for elliptic systems, harmonic maps and minimal graphs*, Edizioni della Normale, Pisa (2005).

[3] *Classification of solutions to the higher order Liouville's equation on* $\mathbb{R}^{2m}$, Math. Z. **263** (2009), 307-329.

[4] *Conformal metrics on* $\mathbb{R}^{2m}$ *with constant* Q*-curvature*, Rend. Lincei. Mat. Appl. **19** (2008), 279-292.

[5] *Concentration-compactness phenomena in higher order Liouville's equation*, J. Funct. Anal. **256** (2009), 3743-3771.

[6] *A threshold phenomenon for embeddings of* H_0^m *into Orlicz spaces*, Calc. Var. Partial Differential Equations. **36** (2009), 493-506.

[7] *Concentration-Compactness phenomena in conformal geometry*, Ph.D. Thesis, ETH Zurich (2009).

[8] (With Mircea Petrache) Asymptotics and quantization for a mean-field equation of higher order, Comm. Partial Differential Equations **35** (2010), 1-22.

[9] (With M. Struwe) Quantization for an elliptic equation of order $2m$ with critical exponential non-linearity. To appear in Math. Z. (2010).

[10] (with M. Petrache) *Existence of solutions to a higher dimensional mean-field equation on manifolds*, Manuscripta Math. **133** (2010), 115-130.

[11] *Quantization for the prescribed* Q*-curvature equation on open domains*, Commun. Contemp. Math. **13** (2011), 533-551.

[12] (With L. Ambrosio and G. De Philippis) *Gamma-convergence of nonlocal perimeter functionals.* Manuscripta Math. **134** (2011), 377-403.

[13] (With C. Mantegazza) *A note on quasilinear parabolic equations on manifolds.* To appear in Annali Sc. Norm. Sup. Pisa.

[14] *A note on* n*-axially symmetric harmonic maps minimizing the relaxed energy.* To appear in J. Funct. Anal. (2011).

[15] (With F. De Marchis and A. Malchiodi) *Critical points of the Moser-Trudinger functional.* Preprint (2011).

Fellowships and Research Grants

10/2000 - 09/2004	Scuola Normale Superiore Fellowship, by National contest (ranked 6th in the Science section)
10/2000 - 09/2004	INdAM Fellowship for undergraduate students in mathematics, by National contest (ranked 2nd).
09/2004 - 08/2007	Stanford Graduate Fellowship (dropped when moving to ETH Zurich).
10/2005 - 09/2006	Scholarship of the Graduate School of Mathematics of Zürich (25000 CHF).
04/2008 - 09/2009	ETH Research Grant “TH” no. ETH-02 08-2 (90000 CHF).
02/2010 - 01/2011	Swiss National Foundation fellowship for prospective researchers no. PBEZP2-129520 (42000 CHF).

Invited speaker

— 23/10/2007 – ETH Zurich, weekly seminar of the Analysis group.

— 25/05/2009 – Cergy-Pontoise, conference ”Geometric and nonlinear analysis”.

— 11/06/2009 – Centro De Giorgi, Scuola Normale Superiore di Pisa, research period ”Geometric Flows and Geometric Operators”.

— 01/07/2009 – Centro De Giorgi, Scuola Normale Superiore di Pisa, workshop ”Geometric Flows and Geometric Operators”.

— 04/08/2009 – Mathematisches Forschungsinstitut Oberwolfach, workshop ”Partielle Differentialgleichungen”.

— 17/09/2009 – Freie Universität Berlin, workshop ”Variational Problems of Higher Order in Geometry”.

— 04/11/2009 – Pisa, weekly seminar of Calculus of Variations.

— 05/05/2010 – SISSA, weekly seminar of the Functional Analysis group.

— 15/12/2010 – Pisa, weekly seminar of Calculus of Variations.

— 09/02/2011 – MIT (Boston), weekly geometry seminar.

— 11/02/2011 – MIT (Boston), mini-course on concentration-compactness.

— 16/02/2011 – Rutgers University (New Jersey), weekly non-linear analysis seminar.

— 18/02/2011 – Princeton University (New Jersey), weekly geometry seminar.

— 24/02/2011 – Columbia University (New York), weekly geometry seminar.

— 19-20/05/2011 – Université de Lyon, 4-hour mini-course.

— 02/06/2011 – SISSA (Trieste), conference ”Higher order operators in geometry and physics”.

TEACHING

During my stay at ETH Zurich I had several teaching duties. I *organized* and/or led exercise groups every semester since Spring 2006. I organized the exercises of the following courses:

Fall 2006	**Differential Geometry**	of Prof. D. Christodoulou
Spring 2007	**PDEs in Differential Geometry**	of Prof. D. Salamon
Spring 2008	**Functional Analysis II (PDE)**	of Prof. M. Struwe
Fall 2008	**Analysis I/II**	of Prof. U. Lang
Spring 2009	**Mathematics II**	of Prof. P. Thurnheer

I also led exercise groups (without organizing them) for the following two courses:

Spring 2006	**Partial Differential Equations**	of Prof. Farkas
Fall 2007	**Functional Analysis I**	of Prof. M. Struwe

During the break between semesters I took part as co-examiner to tens of exams on PDEs and Differential Geometry, and I graded hundreds of written exams.

In Summer 2008 I also *organized* the written exams Analysis I/II of Prof. U. Lang and Prof. M. Torrilhon.

EXTRAS

Olympiads for high-school students

— Both in years 1999 and 2000 I ranked 5th in the Italian Mathematics Olympiads

— In year 2000 I ranked in the top-ten of the Italian Physics Olympiads

— In year 2000 I attended the International Mathematics Olympiads in Taejon (South Korea), and the International Physics Olympiads in Leicester (UK) as contenstant.

Hobbies

— Sport: swimming, volleyball, canoeing

— Reading books

— Music: playing guitar and piano, singing

Pisa, June 4, 2011 Luca Martinazzi